CS 497: Geometrix Data Structures	Spring 2002

Lecture 5 — January 29, 2002

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Voronoi Diagram

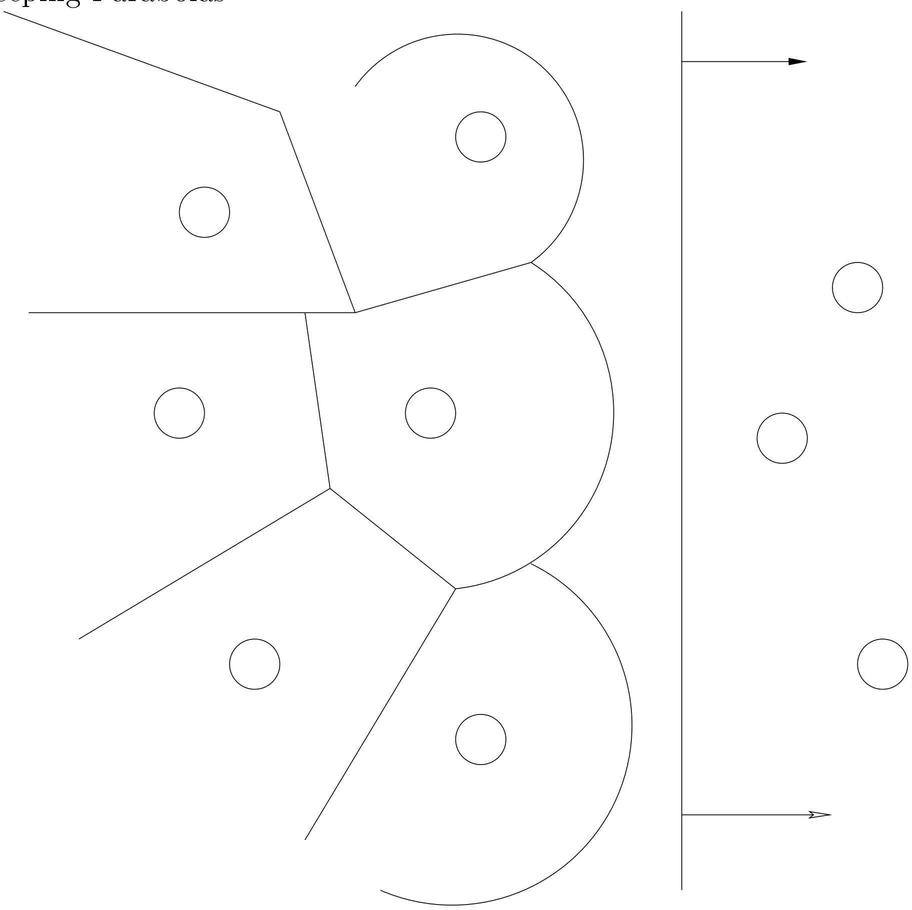
1. Fortune's Sweep

Delannay Triangulations

1. Local Delannayhood
2. Lifting
3. Flipping

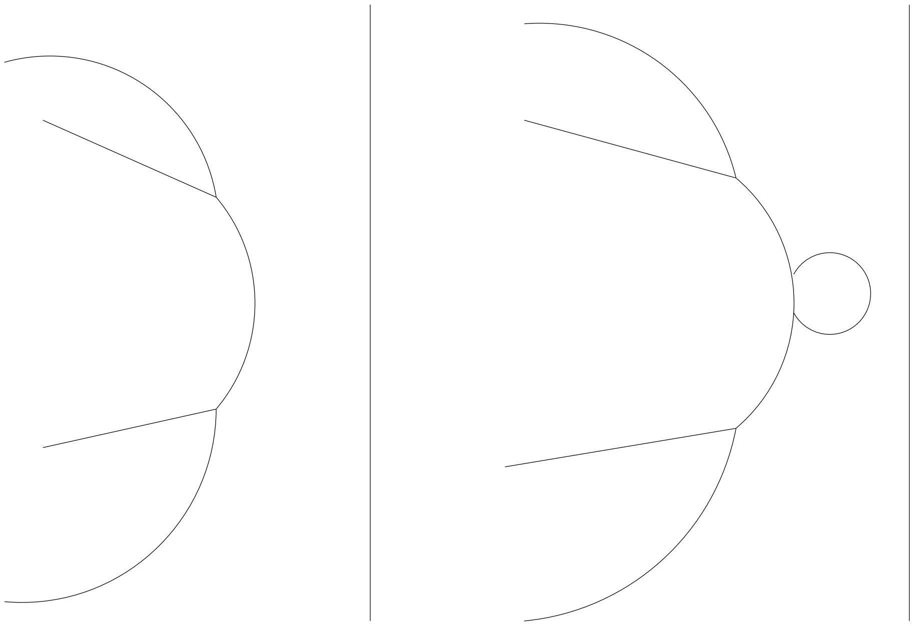
5.1 Fortune's Sweep

Sweeping Parabolas



Wavefront consists of parabolic arcs $\leq 2n-1$
Circle centered on wavefront tangent to sweep line contains ≥ 1 site.
When moving the sweep line, there are two events:

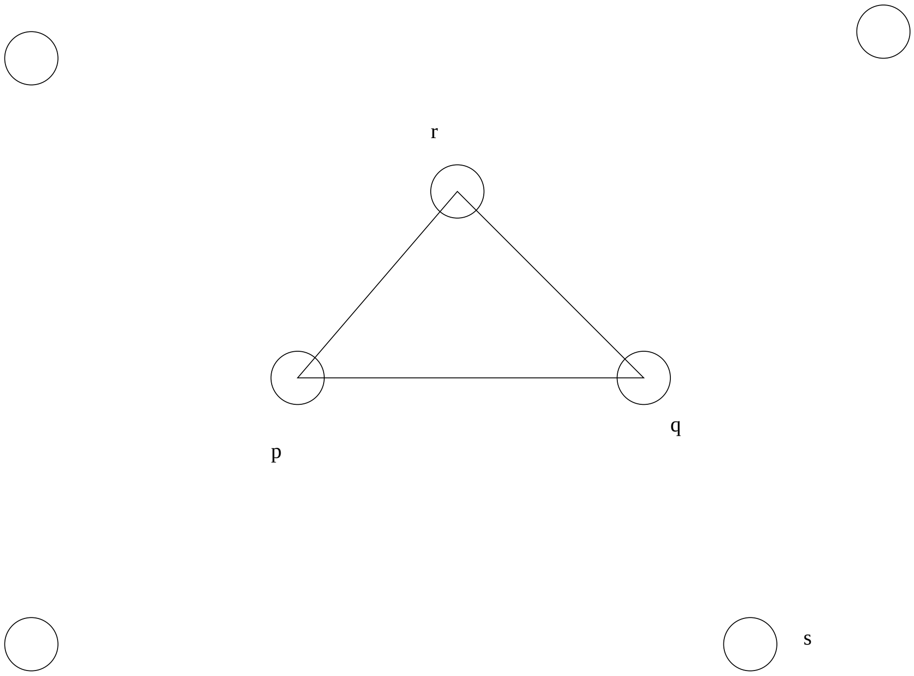
- Birth - when the sweepline passes over a site, one vertex is born.
- Death - when the sweep line hits a circle passing through 3 consecutive sites. This leads to a vernoi vertex.



This algorithm runs in $O(nlog(n)$ time.

5.2 Delannay Triangulation

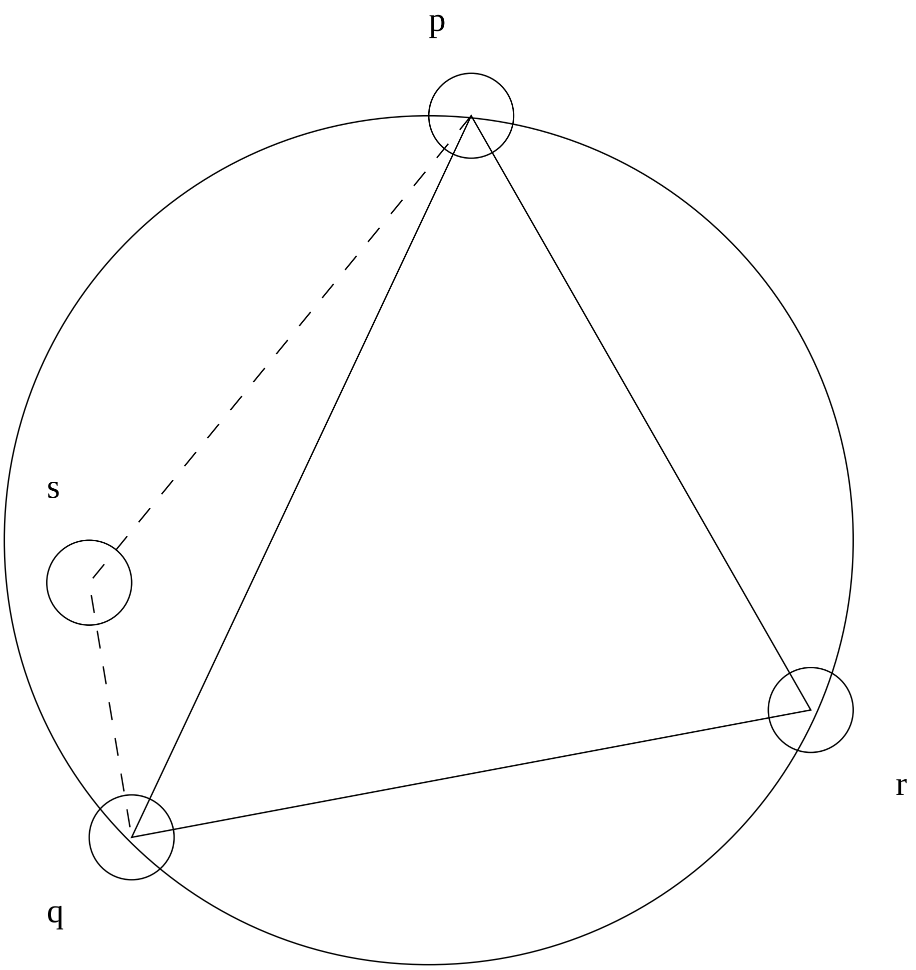
Delannay triangles have empty circumcircles.



An edge pq is locally Delannay if

- pq on a convex hull
- s is outside $O(pqr)$

Theorem: If every edge is locally Delannay, then it is the Delannay triangulation.
Proof: Let T be a non-Delannay triangulation.

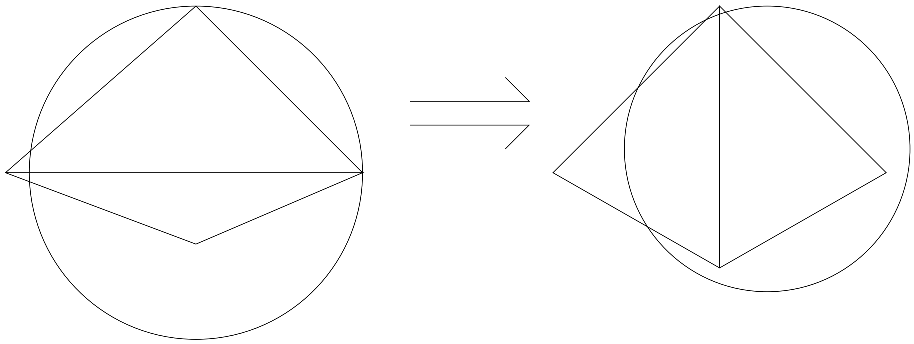


choose pqr and x so that $d(x, pq)$ is minimized.
Either

- pqx is in T implies pq is locally non-Delannay
- otherwise, $pqs \in T$, $x \in Opqs$ and $d(x, ps) < d(x, pq)$ This is a contradiction.

Therefore, only option 1 is possible.

5.2.1 Flipping

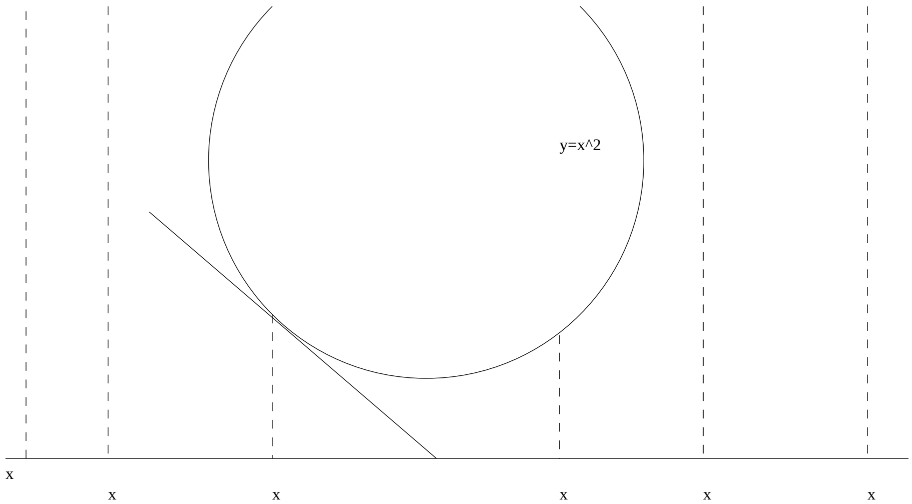


This operation is called a flip.

Theorem: Any triangulation can be transformed into the Delannay triangulation with $O(n^2)$ flips.

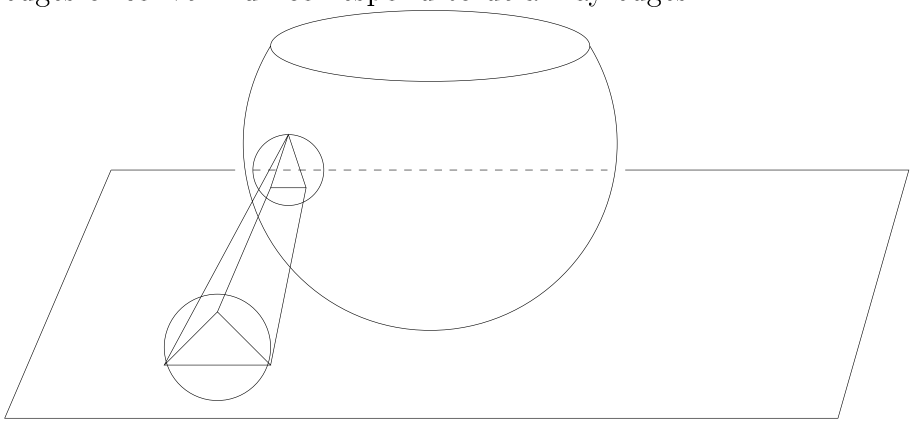
OPEN PROBLEM: is this true is 3D? This problem has been shown to be false in 7D.

5.2.2 Parabaloid Lifting Map



1D delannay is equivalent to sorting.

edges of convex hull correspond to delannay edges.



In like manner we can map a 2D triangulation to a 3D convex hull where each 2D voronoi region implies a facet of upper envelope in 3D.

This implies that Delannay is a dual of Voronoi.