

1. NP-hardness Proofs: Restriction

Prove that each of the following problems is NP-hard. In each part, find a special case of the given problem that is equivalent to a known NP-hard problem.

(a) Longest Path

Given a graph G and a positive integer k , does G contain a path with k or more edges?

(b) Partition into Hamiltonian Subgraphs

Given a graph G and a positive integer k , can the vertices of G be partitioned into at most k disjoint sets such that the graph induced by each set has a Hamiltonian cycle?

(c) Set Packing

Given a collection of finite sets C and a positive integer k , does C contain k disjoint sets?

(d) Largest Common Subgraph

Given two graphs G_1 and G_2 and a positive integer k , does there exist a graph G_3 such that G_3 is a subgraph of both G_1 and G_2 and G_3 has at least k edges?

2. Domino Line

You are given an unusual set of dominoes; each domino has a number on each end, but the numbers may be arbitrarily large and some numbers appear on many dominoes, while other numbers only appear on a few dominoes. Your goal is to form a line using all the dominoes so that adjacent dominoes have the same number on their adjacent halves. Either give an efficient algorithm to solve the problem or show that it is NP-hard.

3. Set Splitting

Given a finite set S and a collection of subsets C is there a partition of S into two sets S_1 and S_2 such that no subset in C is contained entirely in S_1 or S_2 ? Show that the problem is NP-hard. (Hint: use NAE-3SAT, which is similar to 3SAT except that a satisfying assignment does not allow all 3 variables in a clause to be true.)