1. **Max-Flow with vertex capacities**

   In a standard $s - t$ Maximum-Flow Problem, we assume edges have capacities, and there is no limit on how much flow is allowed to pass through a node. In this problem, we consider the variant of Maximum-Flow and Minimum-Cut problems with node capacities.

   More specifically, each node, n_i, has a capacity c_i. The edges have unlimited capacity. Show how you can model this problem as a standard Max-flow problem (where the weights are on the edges).

2. **Emergency evacuation**

   Due to large-scale flooding in a region, paramedics have identified a set of n injured people distributed across the region who need to be reushed to hospitals. There are k hospitals in the region, and each of the n people needs to be brought to a hospital that is within a half-hour's driving time of their current location.

   At the same time, we don't want to overload any hospital by sending too many patients its way. We'd like to distribute the people so that each hospital receives at most $\lceil n/k \rceil$ people.

   Show how to model this problem as a Max-flow problem.

3. **Tracking a Hacker**

   A computer network (with each edge weight 1) is designed to carry traffic from a source s to a destination t. Recently, a computer hacker destroyed some of the edges in the graph. Normally, the maximum $s - t$ flow in G is k. Unfortunately, there is currently no path from s to t. Fortunately, the sysadmins know that the hacker destroyed at most k edges of the graph.

   The sysadmins are trying to diagnose which of the nodes of the graph are no longer reachable. They would like to avoid testing each node. They are using a monitoring tool with the following behavior. If you use the command *ping(v)*, for a given node v, it will tell you whether there is currently a path from s to v (so *ping(t)* will return `False` but *ping(s)* will return `True`).

   Give an algorithm that accomplishes this task using only $O(k \log n)$ pings. (You may assume that any algorithm you wish to run on the original network (before the hacker destroyed edges) runs for free, since you have a model of that network on your computer.)