

CS 473U: Undergraduate Algorithms, Fall 2006

Homework 6

Due Wednesday, November 8, 2006 in 3229 Siebel Center

Remember to turn in separate, individually stapled solutions to each of the problems.

1. Dijkstra's algorithm can be used to determine shortest paths on graphs with some negative edge weights (as long as there are no negative cycles), but the worst-case running time is much worse than the $O(E + V \log V)$ it takes when the edge weights are all positive. Construct an infinite family of graphs - with negative edge weights - for which the asymptotic running time of Dijkstra's algorithm is $\Omega(2^{|V|})$.

2. It's a cold and rainy night, and you have to get home from Siebel Center. Your car has broken down, and it's too windy to walk, which means you have to take a bus. To make matters worse, there is no bus that goes directly from Siebel Center to your apartment, so you have to change buses some number of times on your way home. Since it's cold outside, you want to spend as little time as possible waiting in bus shelters.

From a computer in Siebel Center, you can access an online copy of the MTD bus schedule, which lists bus routes and the arrival time of every bus at each stop on its route. Describe an algorithm which, given the schedule, finds a way for you to get home that minimizes the time you spend at bus shelters (the amount of time you spend on the bus doesn't matter). Since Siebel Center is warm and the nearest bus stop is right outside, you can assume that you wait inside Siebel until the first bus you want to take arrives outside. Analyze the efficiency of your algorithm and prove that it is correct.

3. The Floyd-Warshall all-pairs shortest path algorithm computes, for each $u, v \in V$, the shortest path from u to v . However, if the graph has negative cycles, the algorithm fails. Describe a modified version of the algorithm (*with the same asymptotic time complexity*) that correctly returns shortest-path distances, even if the graph contains negative cycles. That is, if there is a path from u to some negative cycle, and a path from that cycle to v , the algorithm should output $dist(u, v) = -\infty$. For any other pair u, v , the algorithm should output the length of the shortest directed path from u to v .