

## CS 473: Undergraduate Algorithms, Fall 2012

# Headbanging 0: Induction!

August 28 and 29

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1. Prove that any non-negative integer can be represented as the sum of distinct powers of 2. (“Write it in binary” is not a proof; it’s just a restatement of what you have to prove.)
  2. Prove that any integer can be represented as the sum of distinct powers of  $-2$ .
  3. Write *four different* proofs that any  $n$ -node tree has exactly  $n - 1$  edges.
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### Take-home points:

- Induction is recursion. Recursion is induction.
- All induction is strong/structural induction. There is absolutely no point in using a *weak* induction hypothesis. None. Ever.
- To prove that all snarks are boojums, start with an *arbitrary* snark and remove some tentacles. Do not start with a smaller snark and try to add tentacles. Snarks don’t like that.
- Every induction proof requires an exhaustive case analysis. Write down the cases. Make sure they’re exhaustive.
- Do the most general cases first. Whatever is left over are the base cases.
- The empty set is the best base case.

*Khelm is Warsaw. Warsaw is Khelm. Khelm is Warsaw. Zay gezunt!  
Warsaw is Khelm. Khelm is Warsaw. Warsaw is Khelm. For gezunt!*

— Golem (feat. Amanda Palmer), “Warsaw is Khelm”, *Fresh Off Boat* (2006)