

1. Suppose you are given the following information:

- A directed graph  $G = (V, E)$ .
- Two vertices  $s$  and  $t$  in  $V$ .
- A positive edge capacity function  $c: E \rightarrow \mathbb{R}^+$ .
- Another function  $f: E \rightarrow \mathbb{R}$

Describe and analyze an algorithm to determine whether  $f$  is a maximum  $(s, t)$ -flow in  $G$ .

2. Describe an efficient algorithm to determine whether a given flow network contains a *unique* maximum flow.

3. Suppose that a flow network has vertex capacities in addition to edge capacities. That is, the total amount of flow into or out of any vertex  $v$  is at most the capacity of  $v$ :

$$\sum_u f(u \rightarrow v) = \sum_w f(v \rightarrow w) \leq c(v)$$

Describe and analyze an algorithm to compute maximum flows with this additional constraint.