

1. The UIUC Faculty Senate has decided to convene a committee to determine whether Chief Illiniwek should become the official *mascot symbol* of the University of Illinois Global Campus.<sup>1</sup> Exactly one faculty member must be chosen from each academic department to serve on this committee. Some faculty members have appointments in multiple departments, but each committee member will represent only one department. For example, if Prof. Blagojevich is affiliated with both the Department of Corruption and the Department of Stupidity, and he is chosen as the Stupidity representative, then someone else must represent Corruption. Finally, University policy requires that any committee on virtual *mascots symbols* must contain the same number of assistant professors, associate professors, and full professors. Fortunately, the number of departments is a multiple of 3.

Describe an efficient algorithm to select the membership of the Global Illiniwek Committee. Your input is a list of all UIUC faculty members, their ranks (assistant, associate, or full), and their departmental affiliation(s). There are  $n$  faculty members and  $3k$  departments.

2. *Ad-hoc networks* are made up of low-powered wireless devices. In principle<sup>2</sup>, these networks can be used on battlefields, in regions that have recently suffered from natural disasters, and in other hard-to-reach areas. The idea is that a large collection of cheap, simple devices could be distributed through the area of interest (for example, by dropping them from an airplane); the devices would then automatically configure themselves into a functioning wireless network.

These devices can communicate only within a limited range. We assume all the devices are identical; there is a distance  $D$  such that two devices can communicate if and only if the distance between them is at most  $D$ .

We would like our ad-hoc network to be reliable, but because the devices are cheap and low-powered, they frequently fail. If a device detects that it is likely to fail, it should transmit its information to some other *backup* device within its communication range. We require each device  $x$  to have  $k$  potential backup devices, all within distance  $D$  of  $x$ ; we call these  $k$  devices the *backup set* of  $x$ . Also, we do not want any device to be in the backup set of too many other devices; otherwise, a single failure might affect a large fraction of the network.

So suppose we are given the communication radius  $D$ , parameters  $b$  and  $k$ , and an array  $d[1..n, 1..n]$  of distances, where  $d[i, j]$  is the distance between device  $i$  and device  $j$ . Describe an algorithm that either computes a backup set of size  $k$  for each of the  $n$  devices, such that no device appears in more than  $b$  backup sets, or reports (correctly) that no good collection of backup sets exists.

3. Given an undirected graph  $G = (V, E)$ , with three vertices  $u$ ,  $v$ , and  $w$ , describe and analyze an algorithm to determine whether there is a path from  $u$  to  $w$  that passes through  $v$ .

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<sup>1</sup>Thankfully, the Global Campus has faded into well-deserved obscurity, thanks in part to the 2009 admissions scandal. Imagine MOOCs, but with the same business model and faculty oversight as the University of Phoenix.

<sup>2</sup>but not really in practice