**A useful list of NP-hard problems appears on the next page.**

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The KNAPSACK problem is the following. We are given a set of $n$ objects, each with a positive integer ***size*** and a positive integer *value*; we are also given a positive integer $B$. The problem is to choose a subset of the $n$ objects with maximum total value, whose total size is at most $B$. Let $V$ denote the sum of the values of all objects.

1. Describe an algorithm to solve KNAPSACK in time polynomial in $n$ and $V$.

2. Prove that the KNAPSACK problem is NP-hard.

   Given the algorithm from problem 1, why doesn't this immediately imply that P=NP?

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3. ***Facility location*** is a family of problems that require choosing a subset of facilities (for example, gas stations, cell towers, garbage dumps, Starbuckses, . . . ) to serve a given set of locations cheaply. In its most abstract formulation, the input to the facility location problem is a pair of arrays $Open[1..n]$ and $Connect[1..n, 1..m]$, where

   - $Open[i]$ is the cost of opening a facility $i$, and
   - $Connect[i, j]$ is the cost of connecting facility $i$ to location $j$.

   Given these two arrays, the problem is to compute a subset $I \subseteq \{1, 2, \ldots, n\}$ of the $n$ facilities to open and a function $\phi : \{1, 2, \ldots, m\} \to I$ that assigns an open facility to each of the $m$ locations, so that the total cost

   $$\sum_{i \in I} Open[i] + \sum_{j=1}^{m} Connect[\phi(j), j]$$

   is minimized. Prove that this problem is NP-hard.

**You may assume the following problems are NP-hard:**

**CircuitSat:** Given a boolean circuit, are there any input values that make the circuit output True?

**PlanarCircuitSat:** Given a boolean circuit drawn in the plane so that no two wires cross, are there any input values that make the circuit output True?

**3Sat:** Given a boolean formula in conjunctive normal form, with exactly three literals per clause, does the formula have a satisfying assignment?

**Max2Sat:** Given a boolean formula in conjunctive normal form, with exactly two literals per clause, what is the largest number of clauses that can be satisfied by an assignment?

**MaxIndependentSet:** Given an undirected graph $G$, what is the size of the largest subset of vertices in $G$ that have no edges among them?

**MaxClique:** Given an undirected graph $G$, what is the size of the largest complete subgraph of $G$?

**MinVertexCover:** Given an undirected graph $G$, what is the size of the smallest subset of vertices that touch every edge in $G$?

**MinSetCover:** Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set $S$, what is the size of the smallest subcollection whose union is $S$?

**MinHittingSet:** Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set $S$, what is the size of the smallest subset of $S$ that intersects every subset $S_i$?

**3Color:** Given an undirected graph $G$, can its vertices be colored with three colors, so that every edge touches vertices with two different colors?

**MaxCut:** Given a graph $G$, what is the size (number of edges) of the largest bipartite subgraph of $G$?

**HamiltonianCycle:** Given a graph $G$, is there a cycle in $G$ that visits every vertex exactly once?

**HamiltonianPath:** Given a graph $G$, is there a path in $G$ that visits every vertex exactly once?

**TravelingSalesman:** Given a graph $G$ with weighted edges, what is the minimum total weight of any Hamiltonian path/cycle in $G$?

**SubsetSum:** Given a set $X$ of positive integers and an integer $k$, does $X$ have a subset whose elements sum to $k$?

**Partition:** Given a set $X$ of positive integers, can $X$ be partitioned into two subsets with the same sum?

**3Partition:** Given a set $X$ of $n$ positive integers, can $X$ be partitioned into $n/3$ three-element subsets, all with the same sum?