

1. **Snakes and Ladders** is a classic board game, originating in India no later than the 16th century. The board consists of an $n \times n$ grid of squares, numbered consecutively from 1 to n^2 , starting in the bottom left corner and proceeding row by row from bottom to top, with rows alternating to the left and right. Certain pairs of squares in this grid, always in different rows, are connected by either “snakes” (leading down) or “ladders” (leading up). Each square can be an endpoint of at most one snake or ladder.

100	99	98	97	96	95	94	93	92	91
81	82	83	84	85	86	87	88	89	90
80	79	78	77	76	75	74	73	72	71
61	62	63	64	65	66	67	68	69	70
60	59	58	57	56	55	54	53	52	51
41	42	43	44	45	46	47	48	49	50
40	39	38	37	36	35	34	33	32	31
21	22	23	24	25	26	27	28	29	30
20	19	18	17	16	15	14	13	12	11
1	2	3	4	5	6	7	8	9	10

A typical Snakes and Ladders board.

Upward straight arrows are ladders; downward wavy arrows are snakes.

You start with a token in cell 1, in the bottom left corner. In each move, you advance your token up to k positions, for some fixed constant k . If the token ends the move at the *top* end of a snake, it slides down to the bottom of that snake. Similarly, if the token ends the move at the *bottom* end of a ladder, it climbs up to the top of that ladder.

Describe and analyze an algorithm to compute the smallest number of moves required for the token to reach the last square of the grid.

2. Suppose you are given a set of n jobs, indexed from 1 to n , together with a list of *precedence constraints*. Each precedence constraint is a pair (i, j) , indicating that job i must be finished before job j begins. Describe and analyze an algorithm that either finds a schedule for executing all n jobs on a single processor, such that all precedence constraints are satisfied, or correctly reports that no such schedule is possible.
3. For the previous problem, describe and analyze an algorithm to determine whether there is a *unique* schedule for a given number of jobs that satisfies a given set of precedence constraints.