

1. An *American graph* is a directed graph with each vertex colored *red*, *white*, or *blue*. An *American Hamiltonian path* is a Hamiltonian path that cycles between red, white, and blue vertices; that is, every edge goes from red to white, or white to blue, or blue to red. The AMERICANHAMILTONIANPATH problem asks whether there is an American Hamiltonian path in an American graph.
 - (a) Prove that AMERICANHAMILTONIANPATH is NP-complete by reducing from HAMILTONIANPATH.
 - (b) In the opposite direction, reduce AMERICANHAMILTONIANPATH to HAMILTONIANPATH.

2. Given a graph G , the DEG17SPANNINGTREE problem asks whether G has a spanning tree in which each vertex of the spanning tree has degree at most 17. Prove that DEG17SPANNINGTREE is NP-complete.

3. Two graphs are *isomorphic* if one can be transformed into the other by relabeling the vertices. Consider the following related decision problems:
 - GRAPHISOMORPHISM: Given two graphs G and H , determine whether G and H are isomorphic.
 - EVENGRAPHISOMORPHISM: Given two graphs G and H , such that every vertex of G and H have even degree, determine whether G and H are isomorphic.
 - SUBGRAPHISOMORPHISM: Given two graphs G and H , determine whether G is isomorphic to a subgraph of H .
 - (a) Describe a polynomial time reduction from GRAPHISOMORPHISM to EVENGRAPHISOMORPHISM.
 - (b) Describe a polynomial time reduction from GRAPHISOMORPHISM to SUBGRAPHISOMORPHISM.