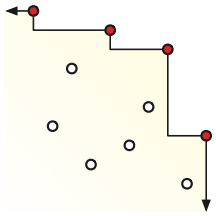
1. What is the *exact* expected number of leaves in a treap with n nodes?

2. Recall question 5 from Midterm 1:

   > Suppose you are given a set P of n points in the plane. A point $p \in P$ is *maximal* in P if no other point in P is both above and to the right of P. Intuitively, the maximal points define a "staircase" with all the other points of P below it.
   >
   > 
   >
   > A set of ten points, four of which are maximal.
   >
   > Describe and analyze an algorithm to compute the number of maximal points in P in $O(n \log n)$ time. For example, given the ten points shown above, your algorithm should return the integer 4.

   Suppose the points in P are generated independently and uniformly at random in the unit square $[0,1]^2$. What is the *exact* expected number of maximal points in P?

3. Suppose you want to write an app for your new Pebble smart watch that monitors the global Twitter stream and selects a small sample of *random* tweets. You will not know when the stream ends until your app attempts to read the next tweet and receives the error message FAILWHALE. The Pebble has only a small amount of memory, far too little to store the entire stream.

   (a) Describe an algorithm that, as soon as the stream ends, returns a single tweet chosen uniformly at random from the stream. Prove your algorithm is correct. (You may assume that the stream contains at least one tweet.)

   (b) Now fix an arbitrary positive integer k. Describe an algorithm that picks k tweets uniformly at random from the stream. Prove your algorithm is correct. (You may assume that the stream contains at least k tweets.)