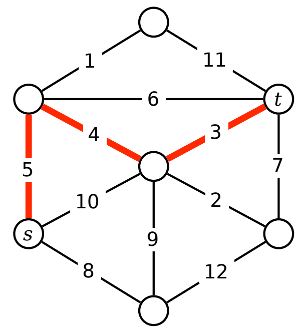
1. Consider a path between two vertices s and t in an undirected weighted graph G. The *bottleneck length* of this path is the maximum weight of any edge in the path. The *bottleneck distance* between s and t is the minimum bottleneck length of any path from s to t. (If there are no paths from s to t, the bottleneck distance between s and t is ∞.)

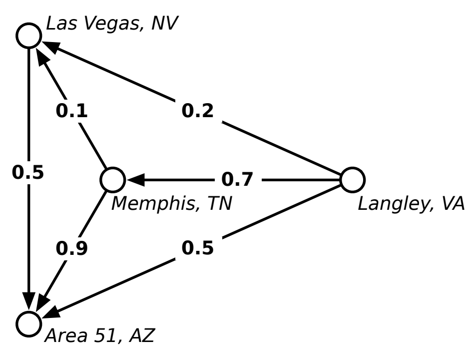


   Describe an algorithm to compute the bottleneck distance between *every* pair of vertices in an arbitrary undirected weighted graph. Assume that no two edges have the same weight.

2. Let G be a directed graph with (possibly negative) edge weights, and let s be an arbitrary vertex of G. Suppose for each vertex v we are given a real number $d(v)$. Describe and analyze an algorithm to determine whether the numbers $d(v)$ on vertices are the shortest path distances from s to each vertex v. Do not assume that the graph G has no negative cycles.

3. Mulder and Scully have computed, for every road in the United States, the exact probability that someone driving on that road *won't* be abducted by aliens. Agent Mulder needs to drive from Langley, Virginia to Area 51, Nevada. What rout should hs take so that he has the least chance of being abducted?

   More formally, you are given a directed graph G, possibly with cycles, where every edge e has an independent safety probability $p(e)$. The *safety* of a path is the product of the safety probabilities of its edges. Design and analyze an algorithm to determine the safest path from a given start vertex s to a given target vertex t.



   For example, with the probabilities shown above, if Mulder tries to drive directly from Langley to Area 51, he has a 50% chance of getting there without being abducted. If he stops in Memphis, he has a $0.7 \times 0.9 = 63\%$ chance of arriving safely. If he stops first in Memphis and then in Las Vegas, he has a $1 - 0.7 \times 0.1 \times 0.5 = 96.5\%$ chance of begin abducted![1]

[1]That's how they got Elvis, you know.