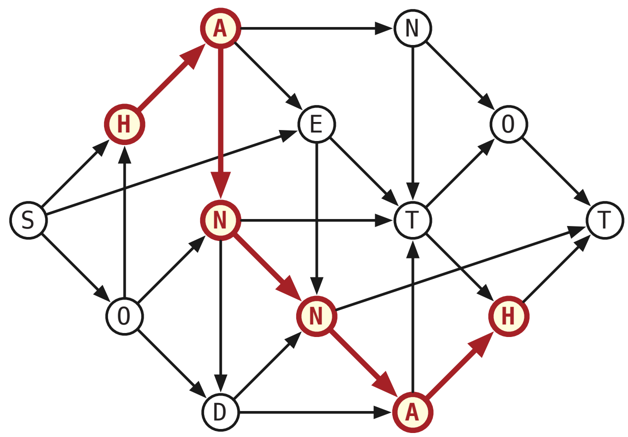
1. Suppose we are given a directed acyclic graph G with labeled vertices. Every path in G has a label, which is a string obtained by concatenating the labels of its vertices in order. Recall that a *palindrome* is a string that is equal to its reversal.

   Describe and analyze an algorithm to find the length of the longest palindrome that is the label of a path in G. For example, given the graph below, your algorithm should return the integer 6, which is the length of the palindrome **HANNAH**.



2. Let G be a connected directed graph that contains both directions of every edge; that is, if $u \to v$ is an edge in G, its reversal $v \to u$ is also an edge in G. Consider the following non-standard traversal algorithm.

```
SpaghettiTraversal(G):
    for all vertices v in G
        unmark v
    for all edges u→v in G
        color u→v white
    s ← any vertex in G
    Spaghetti(s)
```

```
Spaghetti(v):
    mark v                                  ⟨⟨"visit v"⟩⟩
    if there is a white arc v→w
        if w is unmarked
            color w→v green
        color v→w red                       ⟨⟨"traverse v→w"⟩⟩
        Spaghetti(w)
    else if there is a green arc v→w
        color v→w red                       ⟨⟨"traverse v→w"⟩⟩
        Spaghetti(w)
    ⟨⟨else every arc v→w is red, so halt⟩⟩
```

   We informally say that this algorithm "visits" vertex v every time it marks v, and it "traverses" edge $v \to w$ when it colors that edge red. Unlike our standard graph-traversal algorithms, Spaghetti may (in fact, *will*) mark/visit each vertex more than once.

   The following series of exercises leads to a proof that Spaghetti traverses each directed edge of G exactly once. Most of the solutions are very short.

   (a) Prove that no directed edge in G is traversed more than once.

   (b) When the algorithm visits a vertex v for the kth time, exactly how many edges into v are red, and exactly how many edges out of v are red? *[Hint: Consider the starting vertex s separately from the other vertices.]*

(c) Prove each vertex v is visited at most $\deg(v)$ times, except the starting vertex s, which is visited at most $\deg(s)+1$ times. This claim immediately implies that SPAGHETTITRAVERSAL(G) terminates.

(d) Prove that when SPAGHETTITRAVERSAL(G) ends, the last visited vertex is the starting vertex s.

(e) For every vertex v that SPAGHETTITRAVERSAL(G) visits, prove that all edges incident to v (either in or out) are red when SPAGHETTITRAVERSAL(G) halts. *[Hint: Consider the vertices in the order that they are marked for the first time, starting with s, and prove the claim by induction.]*

(f) Prove that SPAGHETTITRAVERSAL(G) visits every vertex of G.

(g) Finally, prove that SPAGHETTITRAVERSAL(G) traverses every edge of G exactly once.