1. Prove that every non-negative integer can be represented as the sum of distinct powers of 2. ("Write it in binary" is not a proof; it's just a restatement of what you have to prove.)

2. Suppose you and your 8-year-old cousin Elmo decide to play a game with a rectangular bar of chocolate, which has been scored into an $n \times m$ grid of squares. You and Elmo alternate turns. On each turn, you or Elmo choose one of the available pieces of chocolate and break it along one of the grid lines into two smaller rectangles. Thus, at all times, each piece of chocolate is an $a \times b$ rectangle for some positive integers a and b; in particular, a 1×1 piece cannot be broken into smaller pieces. The game ends when all the pieces are individual squares. The winner is the player who breaks the last piece.

   Describe a strategy for winning this game. When should you take the first move, and when should you offer it to Elmo? On each turn, how do you decide which piece to break and where? Prove your answers are correct. *[Hint: Let's play a 3 × 3 game. You go first. Oh, and I'm kinda busy right now, so could you just play for me whenever it's my turn? Thanks.]*

3. **[To think about later]** Now consider a variant of the previous chocolate-bar game, where on each turn you can *either* break a piece into two smaller pieces *or eat a* 1×1 *piece*. This game ends when all the chocolate is gone. The winner is the player who eats the last bite of chocolate (*not* the player who eats the *most* chocolate). Describe a strategy for winning this game, and prove that your strategy works.