

For each of the problems below, transform the input into a graph and apply a standard graph algorithm that you’ve seen in class. Whenever you use a standard graph algorithm, you *must* provide the following information. (I recommend actually using a bulleted list.)

- What are the vertices?
  - What are the edges? Are they directed or undirected?
  - If the vertices and/or edges have associated values, what are they?
  - What problem do you need to solve on this graph?
  - What standard algorithm are you using to solve that problem?
  - What is the running time of your entire algorithm, *including* the time to build the graph, *as a function of the original input parameters*?
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1. Inspired by the previous lab, you decided to organize a Snakes and Ladders competition with  $n$  participants. In this competition, each game of Snakes and Ladders involves three players. After the game is finished, they are ranked first, second and third. Each player may be involved in any (non-negative) number of games, and the number needs not be equal among players.

At the end of the competition,  $m$  games have been played. You realized that you had forgotten to implement a proper rating system, and therefore decided to produce the overall ranking of all  $n$  players as you see fit. However, to avoid being too suspicious, if player  $A$  ranked better than player  $B$  in any game, then  $A$  must rank better than  $B$  in the overall ranking.

You are given the list of players involved and the ranking in each of the  $m$  games. Describe and analyze an algorithm to produce an overall ranking of the  $n$  players that satisfies the condition, or correctly reports that it is impossible.

2. There are  $n$  galaxies connected by  $m$  intergalactic teleport-ways. Each teleport-way joins two galaxies and can be traversed in both directions. Also, each teleport-way  $e$  has an associated toll of  $c_e$  dollars, where  $c_e$  is a positive integer. A teleport-way can be used multiple times, but the toll must be paid every time it is used.

Judy wants to travel from galaxy  $u$  to galaxy  $v$ , but teleportation is not very pleasant and she would like to minimize the number of times she needs to teleport. However, she wants the total cost to be a multiple of five dollars, because carrying small bills is not pleasant either.

- (a) Describe and analyze an algorithm to compute the smallest number of times Judy needs to teleport to travel from galaxy  $u$  to galaxy  $v$  while the total cost is a multiple of five dollars.
- (b) Solve (a), but now assume that Judy has a coupon that allows her to waive the toll once.