

Suppose we are given both an undirected graph  $G$  with weighted edges and a minimum spanning tree  $T$  of  $G$ .

1. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge  $e \in T$  is decreased.
2. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge  $e \notin T$  is increased.
3. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge  $e \in T$  is increased.
4. Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge  $e \notin T$  is decreased.

In all cases, the input to your algorithm is the edge  $e$  and its new weight; your algorithms should modify  $T$  so that it is still a minimum spanning tree. Of course, we could just recompute the minimum spanning tree from scratch in  $O(E \log V)$  time, but you can do better.