

Describe how to simulate an arbitrary Turing machine to make it *error-tolerant*. Specifically, given an arbitrary Turing machine M , describe a new Turing machine M' that accepts and rejects exactly the same strings as M , even though an evil pixie named Lenny will move the head of M' to an *arbitrary* location on the tape some finite number of *unknown* times during the execution of M' .

You do not have to describe M' in complete detail, but do give enough details that a seasoned Turing machine programmer could work out the remaining mechanical details.

As stated, this problem has no solution! If M halts on all inputs after a finite number of steps, then Lenny can make any substring of the input string completely invisible to M . For example, if the true input string is **INPUT-STRING**, Lenny can make M believe the input string is actually **IMPING**, by moving the head to the second **I** whenever it tries to move to **R**, and by moving the head to **P** when it tries to move to **U**. Because M halts after a finite number of steps, Lenny only has a finite number of opportunities to move the head.

In fact, with more care, Lenny can make M think the input string is *any* string that uses only symbols from the actual input string; if the true input string is **INPUT-STRING**, Lenny can make M believe the input string is actually **GRINNING-PUTIN-IS-GRINNING**.)

However, there are several different ways to rescue the problem. For each of the following restrictions on Lenny's behavior, and for any Turing machine M , one can design a Turing machine M' that simulates M despite Lenny's interference.

- Lenny can move the head only a *bounded* number of times. For example: Lenny can move the head at most 374 times.
 - Whenever Lenny moves the head, he changes the state of the machine to a special error state **lenny**.
 - Whenever Lenny moves the head, he moves it to the left end of the tape.
 - Whenever Lenny moves the head, he moves it to a blank cell to the right of all non-blank cells.
 - Whenever Lenny moves the head, he moves it to a cell containing a particular symbol in the input alphabet, say **0**.
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