

Prove that the following languages are undecidable *using Rice’s Theorem*:

**Rice’s Theorem.** Let  $\mathcal{X}$  be any nonempty proper subset of the set of acceptable languages. The language  $\text{ACCEPTIN}\mathcal{X} := \{ \langle M \rangle \mid \text{ACCEPT}(M) \in \mathcal{X} \}$  is undecidable.

1.  $\text{ACCEPTREGULAR} := \{ \langle M \rangle \mid \text{ACCEPT}(M) \text{ is regular} \}$
2.  $\text{ACCEPTILLINI} := \{ \langle M \rangle \mid M \text{ accepts the string } \text{ILLINI} \}$
3.  $\text{ACCEPTPALINDROME} := \{ \langle M \rangle \mid M \text{ accepts at least one palindrome} \}$
4.  $\text{ACCEPTTHREE} := \{ \langle M \rangle \mid M \text{ accepts exactly three strings} \}$
5.  $\text{ACCEPTUNDECIDABLE} := \{ \langle M \rangle \mid \text{ACCEPT}(M) \text{ is undecidable} \}$

**To think about later.** Which of the following languages are undecidable? How do you prove it?

1.  $\text{ACCEPT}\{\{\varepsilon\}\} := \{ \langle M \rangle \mid M \text{ only accepts the string } \varepsilon, \text{ i.e. } \text{ACCEPT}(M) = \{\varepsilon\} \}$
2.  $\text{ACCEPT}\{\emptyset\} := \{ \langle M \rangle \mid M \text{ does not accept any strings, i.e. } \text{ACCEPT}(M) = \emptyset \}$
3.  $\text{ACCEPT}\emptyset := \{ \langle M \rangle \mid \text{ACCEPT}(M) \text{ is not an acceptable language} \}$
4.  $\text{ACCEPT}=\text{REJECT} := \{ \langle M \rangle \mid \text{ACCEPT}(M) = \text{REJECT}(M) \}$
5.  $\text{ACCEPT}\neq\text{REJECT} := \{ \langle M \rangle \mid \text{ACCEPT}(M) \neq \text{REJECT}(M) \}$
6.  $\text{ACCEPT}\cup\text{REJECT} := \{ \langle M \rangle \mid \text{ACCEPT}(M) \cup \text{REJECT}(M) = \Sigma^* \}$