Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "True" if the statement is ***always*** true and "False" otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking "I don't know" is worth +¼ point; and flipping a coin is (on average) worth +¼ point. You do ***not*** need to prove your answer is correct.

   Read each statement *very* carefully. Some of these are deliberately subtle.

   (a) If $2+2=5$, then Jeff is not the Queen of England.

   (b) For all languages L, the language L^* is regular.

   (c) For all languages $L \subseteq \Sigma^*$, if L is can be represented by a regular expression, then $\Sigma^* \setminus L$ can also be represented by a regular expression.

   (d) For all languages L_1 and L_2, if L_2 is regular and $L_1 \subseteq L_2$, then L_1 is regular.

   (e) For all languages L_1 and L_2, if L_2 is not regular and $L_1 \subseteq L_2$, then L_1 is not regular.

   (f) For all languages L, if L is not regular, then every fooling set for L is infinite.

   (g) The language $\left\{ \mathtt{0}^m \mathtt{1} \mathtt{0}^n \mid 0 \le n - m \le 374 \right\}$ is regular.

   (h) The language $\left\{ \mathtt{0}^m \mathtt{1} \mathtt{0}^n \mid 0 \le n + m \le 374 \right\}$ is regular.

   (i) For every language L, if L is not regular, then the language $L^R = \left\{ w^R \mid w \in L \right\}$ is also not regular. (Here w^R denotes the reversal of string w; for example, $(\mathtt{BACKWARD})^R = \mathtt{DRAWKCAB}$.)

   (j) Every context-free language is regular.

2. Let L be the set of strings in $\{\mathtt{0}, \mathtt{1}\}^*$ in which every run of consecutive `0`s has odd length and the total number of `1`s is even.

   For example, the string `11110000010111000` is in L, because it has eight `1`s and three runs of consecutive `0`s, with lengths 5, 1, and 3.

   (a) Give a regular expression that represents L.

   (b) Construct a DFA that recognizes L.

   You do ***not*** need to prove that your answers are correct.

3. For each of the following languages over the alphabet $\{\mathtt{0}, \mathtt{1}\}$, either ***prove*** that the language is regular or ***prove*** that the language is not regular. ***Exactly one of these two languages is regular.***

   (a) The set of all strings in which the substrings `10` and `01` appear the same number of times.

   (b) The set of all strings in which the substrings `00` and `01` appear the same number of times.

   For example, both of these languages contain the string `1100001101101`.

4. Consider the following recursive function:

$$odds(w) := \begin{cases} w & \text{if } |w| \le 1 \\ a \cdot odds(x) & \text{if } w = abx \text{ for some } a, b \in \Sigma \text{ and } x \in \Sigma^* \end{cases}$$

Intuitively, *odds* removes every other symbol from the input string, starting with the second symbol. For example, *odds*(0101110) = 0010.

Prove that for any regular language L, the following languages are also regular.

(a) $\textsc{Odds}(L) := \left\{ odds(w) \;\middle|\; w \in L \right\}$.

(b) $\textsc{Odds}^{-1}(L) := \left\{ w \;\middle|\; odds(w) \in L \right\}$.

5. Recall that string concatenation and string reversal are formally defined as follows:

$$w \bullet y := \begin{cases} y & \text{if } w = \varepsilon \\ a \cdot (x \bullet y) & \text{if } w = ax \text{ for some } a \in \Sigma \text{ and } x \in \Sigma^* \end{cases}$$

$$w^R := \begin{cases} \varepsilon & \text{if } w = \varepsilon \\ x^R \bullet a & \text{if } w = ax \text{ for some } a \in \Sigma \text{ and } x \in \Sigma^* \end{cases}$$

Prove that $(w \bullet x)^R = x^R \bullet w^R$, for all strings w and x. Your proof should be complete, concise, formal, and self-contained. You may assume the following identities, which we proved in class:

- $w \bullet (x \bullet y) = (w \bullet x) \bullet y$ for all strings w, x, and y.
- $|w \bullet x| = |w| + |x|$ for all strings w and x.