

**Write your answers in the separate answer booklet.**

Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check “Yes” if the statement is *always* true and “No” otherwise. Each correct answer is worth +1 point; each incorrect answer is worth  $-\frac{1}{2}$  point; checking “I don’t know” is worth  $+\frac{1}{4}$  point; and flipping a coin is (on average) worth  $+\frac{1}{4}$  point. You do *not* need to prove your answer is correct.

**Read each statement very carefully.** Some of these are deliberately subtle.

- (a) If zero is odd, then  $2 + 2 = 5$ .
  - (b) For every language  $L$ , and for every string  $w \in L$ , there is a DFA that accepts  $w$ .
  - (c) Two languages  $L$  and  $L'$  are regular if and only if  $L \cap L'$  is regular.
  - (d) For every language  $L$ , the language  $L^*$  is non-empty.
  - (e) Every regular language is recognized by an NFA with exactly 374 accepting states.
  - (f) If  $L$  does not have a fooling set of size 374, then  $L$  is regular.
  - (g) The language  $\{\emptyset^{374n} \mid n \geq 374\}$  is regular.
  - (h) The language  $\{\emptyset^{37n}1^{4n} \mid n \geq 374\}$  is regular.
  - (i) The language  $\{\emptyset^{3n}1^{74n} \mid n \leq 374\}$  is regular.
  - (j) The empty language is context-free.
2. For any string  $w \in \{0, 1\}^*$ , let  $slash(w)$  be the string in  $\{0, 1, /\}^*$  obtained from  $w$  by inserting a new symbol  $/$  between any two consecutive symbols that are *not* equal. For example:

$$\begin{aligned} slash(\epsilon) &= \epsilon \\ slash(00000) &= 00000 \\ slash(000110111) &= 000/11/0/111 \end{aligned}$$

For any language  $L \subseteq \{0, 1\}^*$ , let  $slash(L) = \{slash(w) \mid w \in L\}$ .

- (a) Draw or describe a DFA that accepts the language  $slash(\{0, 1\}^*)$ .
  - (b) Give a regular expression for the language  $slash(\{0, 1\}^*)$ .
  - (c) **Prove** that for any regular language  $L$ , the language  $slash(L)$  is also regular.
- (You do not need to justify your answers to parts (a) and (b).)

3. Let  $L$  be the language  $\{0^a 1^b 0^c \mid a + b = 2c\}$
- (a) **Prove** that  $L$  is not a regular language.
  - (b) Describe a context-free grammar for  $L$ . (You do not need to justify your answer.)
4. For each of the following languages  $L$ , give a regular expression that represents  $L$  **and** draw or describe a DFA that recognizes  $L$ . You do not need to justify your answers.
- (a) All strings in  $\{0, 1\}^*$  that do not contain either  $001$  or  $110$  as a substring
  - (b) All strings in  $\{0, 1, 2\}^*$  that do not contain either  $01$  or  $12$  as a substring
5. For any string  $w \in \{0, 1\}^*$ , let  $oblivate(w)$  denote the string obtained from  $w$  by removing every  $1$ . For example:

$$oblivate(\varepsilon) = \varepsilon$$

$$oblivate(000000) = 000000$$

$$oblivate(111111) = \varepsilon$$

$$oblivate(0100001101) = 000000$$

Let  $L$  be an arbitrary regular language.

- (a) **Prove** that the language  $\{oblivate(w) \mid w \in L\}$  is regular.
- (b) **Prove** that the language  $\{w \in \{0, 1\}^* \mid oblivate(w) \in L\}$  is regular.