

Describe deterministic finite-state automata that accept each of the following languages over the alphabet  $\Sigma = \{0, 1\}$ . You may find it easier to describe these DFAs formally than to draw pictures.

Either drawings or formal descriptions are acceptable, as long as the states  $Q$ , the start state  $s$ , the accept states  $A$ , and the transition function  $\delta$  are all clear. Try to keep the number of states small.

1. All strings in which the number of 0s is even **and** the number of 1s is *not* divisible by 3.
2. All strings in which the number of 0s is even **or** the number of 1s is *not* divisible by 3.
3. All strings that are **both** the binary representation of an integer divisible by 3 **and** the ternary (base-3) representation of an integer divisible by 4.

For example, the string 1100 is an element of this language, because it represents  $2^3 + 2^2 = 12$  in binary and  $3^3 + 3^2 = 36$  in ternary.

#### Harder problems to think about later:

4. All strings in which the subsequence 0101 appears an even number of times.
5. All strings  $w$  such that  $\binom{|w|}{2} \bmod 6 = 4$ .  
*[Hint: Maintain both  $\binom{|w|}{2} \bmod 6$  and  $|w| \bmod 6$ .]*  
*[Hint:  $\binom{n+1}{2} = \binom{n}{2} + n$ .]*
- \*6. All strings  $w$  such that  $F_{\#(10,w)} \bmod 10 = 4$ , where  $\#(10, w)$  denotes the number of times 10 appears as a substring of  $w$ , and  $F_n$  is the  $n$ th Fibonacci number:

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{otherwise} \end{cases}$$