CS 473: Undergraduate Algorithms, Spring 2009
HBS 10

1. Consider the following problem, called *BOX-DEPTH*: Given a set of n axis-aligned rectangles in the plane, how big is the largest subset of these rectangles that contain a common point?

   (a) Describe a polynomial-time reduction from *BOX-DEPTH* to *MAX-CLIQUE*.

   (b) Describe and analyze a polynomial-time algorithm for *BOX-DEPTH*. [Hint: $O(n^3)$ time should be easy, but $O(n \log n)$ time is possible.]

   (c) Why don't these two results imply that $P = NP$?

2. Suppose you are given a magic black box that can determine in polynomial time, given an arbitrary weighted graph G, the length of the shortest Hamiltonian cycle in G. Describe and analyze a polynomial-time algorithm that computes, given an arbitrary weighted graph G, the shortest Hamiltonian cycle in G, using this magic black box as a subroutine.

3. Prove that the following problems are NP-complete.

   (a) Given an undirected graph G, does G have a spanning tree in which every node has degree at most 17?

   (b) Given an undirected graph G, does G have a spanning tree with at most 42 leaves?