
CS 473: Undergraduate Algorithms, Spring 2009

HBS 4

1. Let x and y be two elements of a set S whose ranks differ by exactly r . Prove that in a treap for S , the expected length of the unique path from x to y is $O(\log r)$

2. Consider the problem of making change for n cents using the least number of coins.
 - (a) Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Prove that your algorithm yields an optimal solution.
 - (b) Suppose that the available coins have the values c^0, c^1, \dots, c^k for some integers $c > 1$ and $k \geq 1$. Show that the greedy algorithm always yields an optimal solution.
 - (c) Give a set of 4 coin values for which the greedy algorithm does not yield an optimal solution, show why.
 - (d) Give a dynamic programming algorithm that yields an optimal solution for an arbitrary set of coin values.

3. A heater is a sort of dual treap, in which the priorities of the nodes are given, but their search keys are generate independently and uniformly from the unit interval $[0,1]$. You can assume all priorities and keys are distinct. Describe algorithms to perform the operations INSERT and DELETEMIN in a heater. What are the expected worst-case running times of your algorithms? In particular, can you express the expected running time of INSERT in terms of the priority rank of the newly inserted item?