CS 473: Undergraduate Algorithms, Spring 2009
HBS 7

1. Let $G = (V, E)$ be a directed graph with non-negative capacities. Give an efficient algorithm to check whether there is a unique max-flow on G?

2. Let $G = (V, E)$ be a graph and $s, t \in V$ be two specific vertices of G. We call $(S, T = V \setminus S)$ an (s, t)-cut if $s \in S$ and $t \in T$. Moreover, it is a minimum cut if the sum of the capacities of the edges that have one endpoint in S and one endpoint in T equals the maximum (s, t)-flow. Show that, both intersection and union of two min-cuts is a min-cut itself.

3. Let $G = (V, E)$ be a graph. For each edge e let $d(e)$ be a demand value attached to it. A flow is feasible if it sends more than $d(e)$ through e. Assume you have an oracle that is capable of solving the maximum flow problem. Give efficient algorithms for the following problems that call the oracle at most once.

   (a) Find a feasible flow.

   (b) Find a feasible flow of minimum possible value.