

CS 473: Undergraduate Algorithms, Spring 2009

Homework 1

Due Tuesday, February 3, 2009 at 11:59:59pm.

- Groups of up to three students may submit a single, common solution for this and all future homeworks. Please clearly write every group member's name and NetID on every page of your submission.

1. The traditional Devonian/Cornish drinking song "The Barley Mow" has the following pseudolyrics, where $container[i]$ is the name of a container that holds 2^i ounces of beer. One version of the song uses the following containers: nipperkin, gill pot, half-pint, pint, quart, pottle, gallon, half-anker, anker, firkin, half-barrel, barrel, hogshead, pipe, well, river, and ocean. (Every container in this list is twice as big as its predecessor, except that a firkin is actually 2.25 ankers, and the last three units are just silly.)

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BARLEYMOW( $n$ ):
  "Here's a health to the barley-mow, my brave boys,"
  "Here's a health to the barley-mow!"

  "We'll drink it out of the jolly brown bowl,"
  "Here's a health to the barley-mow!"
  "Here's a health to the barley-mow, my brave boys,"
  "Here's a health to the barley-mow!"

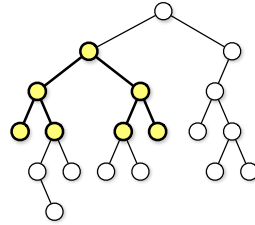
  for  $i \leftarrow 1$  to  $n$ 
    "We'll drink it out of the container[ $i$ ], boys,"
    "Here's a health to the barley-mow!"
    for  $j \leftarrow i$  downto 1
      "The container[ $j$ ],"
      "And the jolly brown bowl!"
      "Here's a health to the barley-mow!"
      "Here's a health to the barley-mow, my brave boys,"
      "Here's a health to the barley-mow!"

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- (a) Suppose each container name $container[i]$ is a single word, and you can sing four words a second. How long would it take you to sing $BARLEYMOW(n)$? (Give a tight asymptotic bound.) [Hint: Is 'barley-mow' one word or two? Does it matter?]
- (b) If you want to sing this song for $n > 20$, you'll have to make up your own container names. To avoid repetition, these names will get progressively longer as n increases¹. Suppose $container[n]$ has $\Theta(\log n)$ syllables, and you can sing six syllables per second. Now how long would it take you to sing $BARLEYMOW(n)$? (Give a tight asymptotic bound.)
- (c) Suppose each time you mention the name of a container, you actually drink the corresponding amount of beer: one ounce for the jolly brown bowl, and 2^i ounces for each $container[i]$. Assuming for purposes of this problem that you are at least 21 years old, *exactly* how many ounces of beer would you drink if you sang $BARLEYMOW(n)$? (Give an *exact* answer, not just an asymptotic bound.)

¹"We'll drink it out of the hemisemidemiyoappint, boys!"

2. For this problem, a *subtree* of a binary tree means any connected subgraph; a binary tree is *complete* if every leaf has exactly the same depth. Describe and analyze a recursive algorithm to compute the *largest complete subtree* of a given binary tree. Your algorithm should return the root and the depth of this subtree.



The largest complete subtree of this binary tree has depth 2.

3. (a) Describe and analyze a recursive algorithm to reconstruct a binary tree, given its preorder and postorder node sequences (as in Homework 0, problem 1).
- (b) Describe and analyze a recursive algorithm to reconstruct a binary tree, given its preorder and *inorder* node sequences.