Rice's Theorem. *Let $\mathcal{L}$ be any set of languages that satisfies the following conditions:*

- *There is a Turing machine Y such that $\text{Accept}(Y) \in \mathcal{L}$.*
- *There is a Turing machine N such that $\text{Accept}(N) \notin \mathcal{L}$.*

The language $\text{AcceptIn}(\mathcal{L}) := \{\langle M\rangle \mid \text{Accept}(M) \in \mathcal{L}\}$ *is undecidable.*

You may find the following Turing machines useful:

- M_{Accept} accepts every input.
- M_{Reject} rejects every input.
- M_{Hang} infinite-loops on every input.

Prove that the following languages are undecidable *using Rice's Theorem*:

1. $\text{AcceptRegular} := \{\langle M\rangle \mid \text{Accept}(M) \text{ is regular}\}$
2. $\text{AcceptIllini} := \{\langle M\rangle \mid M \text{ accepts the string } \texttt{ILLINI}\}$
3. $\text{AcceptPalindrome} := \{\langle M\rangle \mid M \text{ accepts at least one palindrome}\}$
4. $\text{AcceptThree} := \{\langle M\rangle \mid M \text{ accepts exactly three strings}\}$
5. $\text{AcceptUndecidable} := \{\langle M\rangle \mid \text{Accept}(M) \text{ is undecidable}\}$

To think about later. Which of the following languages are undecidable? How would you prove that? Remember that we know several ways to prove undecidability:

- Diagonalization: Assume the language is decidable, and derive an algorithm with self-contradictory behavior.
- Reduction: Assume the language is decidable, and derive an algorithm for a known undecidable language, like Halt or SelfReject or NeverAccept.
- Rice's Theorem: Find an appropriate family of languages $\mathcal{L}$, a machine Y that accepts a language in $\mathcal{L}$, and a machine N that does not accept a language in $\mathcal{L}$.
- Closure: If two languages L and L' are decidable, then the languages $L \cap L'$ and $L \cup L'$ and $L \setminus L'$ and $L \oplus L'$ and L^* are all decidable, too.

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6. $\text{Accept}\{\{\varepsilon\}\} := \{\langle M\rangle \mid M \text{ accepts only the string } \varepsilon\text{; that is, } \text{Accept}(M) = \{\varepsilon\}\}$
7. $\text{Accept}\{\varnothing\} := \{\langle M\rangle \mid M \text{ does not accept any strings; that is, } \text{Accept}(M) = \varnothing\}$
8. $\text{Accept}\varnothing := \{\langle M\rangle \mid \text{Accept}(M) \text{ is not an acceptable language}\}$
9. $\text{Accept=Reject} := \{\langle M\rangle \mid \text{Accept}(M) = \text{Reject}(M)\}$
10. $\text{Accept}{\neq}\text{Reject} := \{\langle M\rangle \mid \text{Accept}(M) \neq \text{Reject}(M)\}$
11. $\text{Accept}{\cup}\text{Reject} := \{\langle M\rangle \mid \text{Accept}(M) \cup \text{Reject}(M) = \Sigma^*\}$