Write your answers in the separate answer booklet.
Please return this question sheet and your cheat sheet with your answers.

1. For each statement below, check "Yes" if the statement is ***always*** true and "No" otherwise. Each correct answer is worth +1 point; each incorrect answer is worth −½ point; checking "I don't know" is worth +¼ point; and flipping a coin is (on average) worth +¼ point. You do ***not*** need to prove your answer is correct.

   Read each statement *very* carefully. Some of these are deliberately subtle.

   (a) No infinite language is regular.

   (b) If L is regular, then for every string $w \in L$, there is a DFA that rejects w.

   (c) If L is context-free and L has a finite fooling set, then L is not regular.

   (d) If L is regular and $L' \cap L = \varnothing$, then L' is not regular.

   (e) The language $\{0^i 1^j 0^k \mid i + j + k \geq 374\}$ is regular.

   (f) The language $\{0^i 1^j 0^k \mid i + j - k \geq 374\}$ is regular.

   (g) Let $M = (Q, \{0, 1\}, s, A, \delta)$ be an arbitrary DFA, and let $M' = (Q, \{0, 1\}, s, A, \delta')$ be the DFA obtained from M by changing every 0-transition into a 1-transition and vice versa. More formally, M and M' have the same states, input alphabet, starting state, and accepting states, but $\delta'(q, 0) = \delta(q, 1)$ and $\delta'(q, 1) = \delta(q, 0)$. Then $L(M) \cup L(M') = \{0, 1\}^*$.

   (h) Let $M = (Q, \Sigma, s, A, \delta)$ be an arbitrary NFA, and $M' = (Q', \Sigma, s, A', \delta')$ be any NFA obtained from M by deleting some subset of the states. More formally, we have $Q' \subseteq Q$, $A' = A \cap Q'$, and $\delta'(q, a) = \delta(q, a) \cap Q'$ for all $q \in Q'$. Then $L(M') \subseteq L(M)$.

   (i) For every non-regular language L, the language $\left\{0^{|w|} \;\middle|\; w \in L\right\}$ is also non-regular.

   (j) For every context-free language L, the language $\left\{0^{|w|} \;\middle|\; w \in L\right\}$ is also context-free.

2. For any language L, define

$$\textsc{StripFinal0s}(L) = \{w \mid w0^n \in L \text{ for some } n \geq 0\}$$

   Less formally, $\textsc{StripFinal0s}(L)$ is the set of all strings obtained by stripping any number of final 0s from strings in L. For example, if L is the one-string language $\{01101000\}$, then

$$\textsc{StripFinal0s}(L) = \{01101,\ 011010,\ 0110100,\ 01101000\}.$$

   Prove that if L is a regular language, then $\textsc{StripFinal0s}(L)$ is also a regular language.

3. For each of the following languages L over the alphabet $\Sigma = \{0, 1\}$, give a regular expression that represents L ***and*** describe a DFA that recognizes L.

   (a) $\left\{0^n w 1^n \mid n \geq 1 \text{ and } w \in \Sigma^+\right\}$

   (b) All strings in $0^*1^*0^*$ whose length is even.

4. The *parity* of a bit-string is 0 if the number of 1 bits is even, and 1 if the number of 1 bits is odd. For example:

$$parity(\varepsilon) = 0 \qquad parity(0010100) = 0 \qquad parity(00101110100) = 1$$

   (a) Give a *self-contained*, formal, recursive definition of the *parity* function. In particular, do ***not*** refer to # or other functions defined in class.

   (b) Let L be an arbitrary regular language. Prove that the language $OddParity(L) := \{w \in L \mid parity(w) = 1\}$ is also regular.

   (c) Let L be an arbitrary regular language. Prove that the language $AddParity(L) := \{parity(w) \cdot w \mid w \in L\}$ is also regular. For example, if L contains the strings 01110 and 01100, then $AddParity(L)$ contains the strings 101110 and 001100.

5. Let L be the language $\left\{0^i 1^j 0^k \mid 2i = k \text{ or } i = 2k\right\}$.

   (a) ***Prove*** that L is not a regular language.

   (b) Describe a context-free grammar for L.