The problem that I am interested in is subquadratic-time algorithms for shortest paths in an arrangement. In this problem, you are given a set of *n* lines in a plane. On those lines are two points, *s* and *t*. The problem is to calculate the shortest path from *s* to *t* that is within the union of lines. This problem has been explored as described below. I would like to explore a variant of it where different regions of the graph have different weights. The purpose of this project is to see how the addition of weight to lines affects the existing algorithms and if it suggests new algorithms that are subquadratic.

This problem is very realistic and practical to study. Imagine that the union of lines was a road map. Then the problem is finding the way to drive from one place to another. Now imagine that we divide the plane into two regions with a single line. This is analogous to the divide between a city and the country. In the country, movement along roads is faster so the lines are weighted less. In the city, movement is slower and the edges have greater cost.

This problem without weights on the lines has been studied with various results over the years. The first and most important result is that this problem can be solved in $O(n^2)$ time. The arrangement of the lines can be computed in $O(n^2)$ time by standardize techniques. This computation leaves you with a planar graph with edge weights. Since the early 90s, there have been shortest path algorithms that work on planar graphs in linear time. One popular algorithm is described in a paper by Henzinger, Klein, and Rao.

The first techniques used to compute these solutions subquadratically are with approximation algorithms. Bose et al. gave a 2-approximation of the problem that runs in $O(n \log n)$ time. Their algorithm runs by running Dijkstra's algorithm on a subgraph of the original graph. This result was improved upon by David Hart in 2002 to a $(1+e)$-approximation with a more complicated, but also subquadratic running time.

The second techniques used to compute these solutions are by restricting the set of lines in some manner. The first paper that I could find that does this is by Eppstein and Hart. They developed an $O(n+k^2)$ algorithm for a set of *n* lines that have *k* different orientations. Another recent paper by Kavitha defined a "pencil" as a set of lines that met at a point. This paper gave an $O(n+m)$ algorithm for finding an *s-t* path within the union of two of these pencils.

The problem that I am interested is applying weights to these lines, seeing which of these algorithms can be stretched to still find the shortest path, and which of these algorithms cannot keep their subquadratic character. With different weighting schemes are there different techniques that need to be used to preserve the correctness of the algorithms. I would like to examine three possible weighting schemes. The first would be a single line dividing the plane into two regions of two different weights. The second would be *k* lines that intersect at a point dividing the plane into *2k* regions of different weight. Finally, I would like to examine a scheme that has a different weight for each different line.

Initial References

J. S. B. Mitchell. Geometric Shortest Paths and Network Optimizations. *Handbook of Computational Geometry*. Elsevier Science Publishers B.V. North-Holland, Amsterdam, 1998.

P. Agarwal and M. Sharir. Arrangements and Their Applications. *Handbook of Computational Geometry*. Elsevier Science Publishers B.V. North-Holland, Amsterdam, 1998.

M. R. Henzinger, P. Klein, and S. Rao. Faster shortest-path algorithms for planar graphs. *J. Comput. Syst. Sci*., 55:3-23, 1997.

P. Bose, W. Evans, D. Kirkpatrick, M. McAllister, and J. Snoeyink. Approximating shortest paths in arrangement of lines. *In Proc. 8th Canadian Conference on Computational Geometry*, pages 143-148. Carleton University Press, Ottawa, Canada, 1996

D. Hart. Approximating the Shortest Path in Line Arrangements. *Proc. Canadian Conference on Computational Geometry.* 2002.

D. Eppstein and D. Hart. Shortest paths in an Arrangement with k Line Orientations. Manuscript, University of California, Irvine, 1997.

T. Kavitha and K. Varadarajan. On Shortest Paths in Line Arrangements. *Canadian Conference on Computational Geometry 2003*: pp.170-173.