Goal: Design or improve algorithms for convex hulls of imprecise points.

In all the problems we have seen so far, our input points have been exact and the only error we could introduce was in computation. This will not always be the case; we might be given points which are subject to measurement error. In this case, we will not have well defined points, but rather a subset of the plane defining possible positions for our points. We'll call each subset an *imprecise point* (sometimes ϵ-point). The choice of model here is not strict, but may be selected to fit our application. For instance if we know that measurement error may occur in only the y-coordinate, we model our imprecise points as a set of parallel vertical lines. For measurement error in both x, y coordinates, our imprecise points are axis-aligned squares. And if we know that the error has magnitude less that ϵ, the imprecise points are circles of radius ϵ. In geometric problems on imprecise points, no matter what the model is, we will always choose one exact point from each imprecise point, and then compute the corresponding problem on the exact points.

If we were now asked to compute the convex hull of a set of imprecise points, we would respond with, "Which one?" There will be a continuum of convex hulls which select one exact point from each imprecise point. So we now need to specify some function of a convex hull and then ask for a hull of the imprecise points which is extremal according to the function. For example, we may ask for the hull of the imprecise points that has maximum or minimum area, or longest or shortest perimeter.

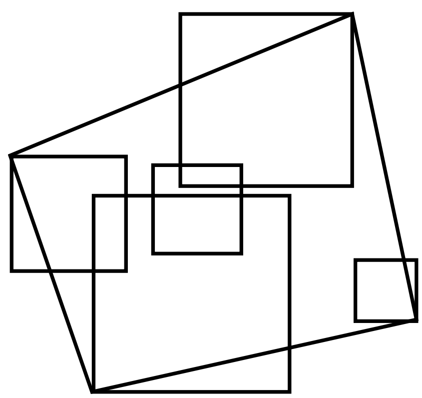

Figure 1. A set of imprecise points modeled as axis-aligned squares and the maximum area convex hull of these points.

Work has been done on finding max/min area and max/min perimeter convex hulls of imprecise points modeled as axis-aligned squares in [2]. The running time of these algorithms, however, suggests plenty of room for improvement (max area convex hull in $O(n^3)$, max perimeter in $O(n^{13})$!). Boissonnat and Lazard [3] show that the problem of finding a convex hull of bounded curvature is equivalent to the imprecise convex hull problem on circles of fixed radius. Unfortunately they give only an exponential algorithm. Nagai and Tokura [1] solve a related problem which is to compute the union and intersection of all possible convex hulls of a set of imprecise points.

References

[1] T. Nagai, N. Tokura, Tight Error Bounds of Geometric Problems on Convex Objects with Imprecise Coordinates. *Jap. Conf. Discr. Comp. Geom.,* 252-263, 2000.

[2] M. Löffler, M. van Kreveld, Largest and Smallest Convex Hulls for Imprecise Points. *Technical Report UU-CS-2006-019, Department of Computing and Information Science,* Utrecht University, 2006.

[3] J.-D. Boissonnat, S. Lazard, Convex Hulls of Bounded Curvature. *Proc. 8th Canad. Conf. on Comput. Geom.,* pages 14-19, 1996.