Geometric spanners

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1 Background

A *spanner* H is a subgraph of a given edge weighted graph G such that $V(H) = V(G)$ and $E(H) \subseteq E(G)$. Typically the size of a spanner is much smaller than the parent graph and the distance $d_H(u, v)$ between any two vertices u, v in a spanner is bounded multiple of the distance $d_G(u, v)$ in the parent subgraph. A *geometric graph* is is a graph such that each of the vertices in a graph is a point in R^d and the distance between two vertices connected by an edge is the length of the edge (in eucleidean norm) in R^d. A spanner defined on a geometric graph is a *geometric spanner*.

2 Open problems

There are several interesting open problems related to geometric spanners a couple of them are mentioned below.

1. To efficiently maintain geometric spanners under insertion and deletion in $\log^c n$ time. In SODA 2008 an algorithm with $O(\log^3 n)$ update time was presented for spanners with constant doubling dimension[1]. More efficient and more general results would be interesting.

2. A geometric spanner is *fault tolerant* if deleting vertices and edges in a region S still leaves the spanner property in the remaining graph intact. Abam et al. showed in a paper in SODA 2007 that when S is convex there exists a fault tolerant t spanner of size $O(n \log n)$. It is unknown whether there exist geometric spanners of linear size.

[1]The doubling dimension of R^d is around $\Theta(d)$.

There are other related problems like bounding the size of skip-list spanners and so on which can also be explored.