CS 598JE Computational Geometry Term Project Proposal

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1 Problem Formulation

Given V, the vertices of Voronoi diagram of an unknown point set P, describe an efficient algorithm to reconstruct a set P (or even better, a description of all such sets) whose Voronoi diagram has V as vertices, or prove that no such algorithm is possible. If necessary, assume that every Voronoi vertex has degree 3.

2 Approaches

2.1 Coincident Rays

The main difficulty of this problem is that the edges of Voronoi diagram are unknown, i.e. among a huge amount of possible "edge configurations" there is one or many that is consistent of input V. With each edge configuration EC, from every vertex $p \in V$ we can draw three rays emanating from p as shown in Figure 1.

Then in each convex polygon bounded by edges of EC we can check whether these rays coincide at one point. If there exist one such polygon in which the rays do not coincide, the corresponding EC cannot be valid, i.e. it is inconsistent.

2.2 Angles

Examining angle relations in Figure 1, we have

$$x = \pi - \angle qps, y = \pi - \angle rps, z = \pi - \angle qpr. \tag{1}$$

This implies that if we modify EC by connecting p with a new vertex q' instead of q, generating a new edge $e(pq')$ which can be regarded as the result of rotating edge $e(pq)$ clockwise plus a scaling, then rays n and l will rotate clockwise, and m will rotate counterclockwise, all with the same amount as $e(pq')$ relative to $e(pq)$. We shall investigate possibility of applying this fact when trying to modify current invalid EC into a valid one.

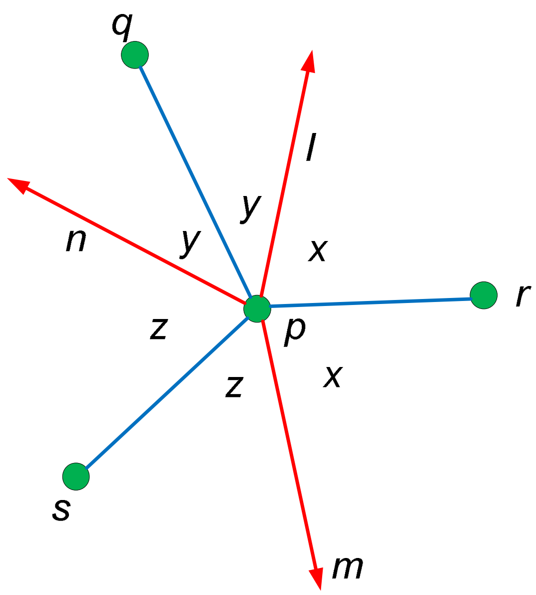

Figure 1: $p, q, r, s \in V$, x, y, z denote angles.