Paper Presentations

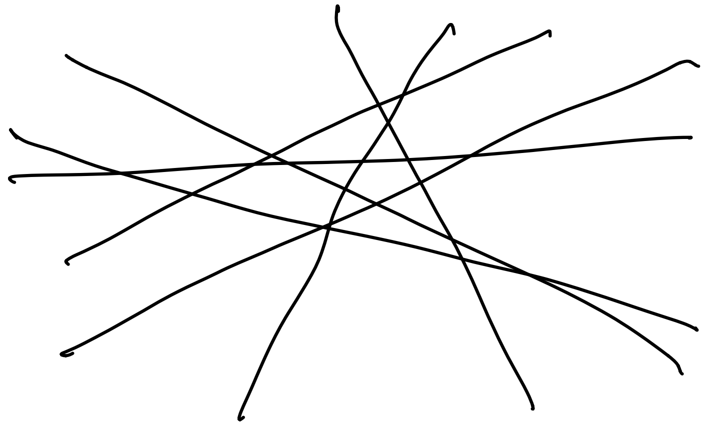
Proposals due tomorrow

This Is It – "Bad Things That Could Happen"
Orkestra Obsolete – "Blue Monday"

Six 20-minute talks

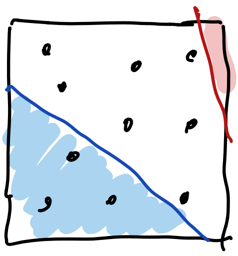
HW 5 due 2 weeks

Applications of Line Arrangements



Arrgh(L)

Discrepancy



Set P of points in $\square = [0,1]^2$

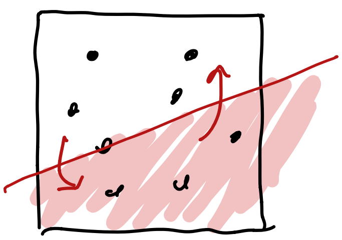
For any halfspace h

$\mu(h) = \text{area}(h \cap \square)$

$\mu_P(h) = |P \cap h| / |P|$

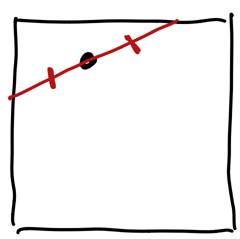
discrepancy of $h = |\mu(h) - \mu_P(h)|$

discrepancy of $P = \max_h |\mu(h) - \mu_P(h)|$



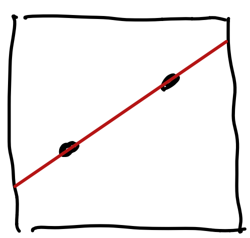
Worst halfspace must have a point on boundary

Extreme lines:



Thru one point at midpoint of $\ell \cap \square$

$O(n)$ such lines
≤ 12 thru any point



Through two points

$\Theta(n^2)$ such lines

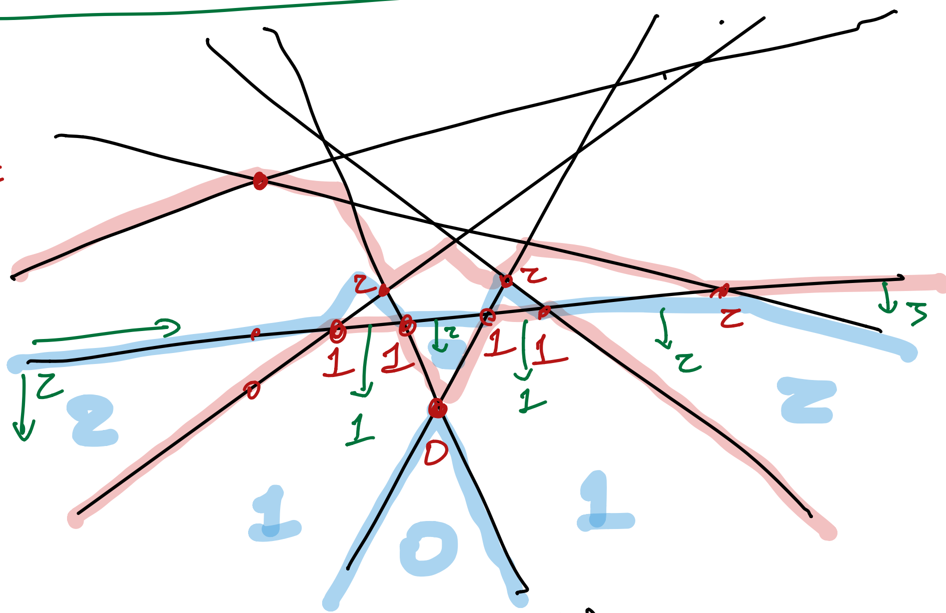
Naively — we can compute disc(h) in $O(n)$ time
disc(P) in $O(n^3)$ time

Bottleneck: $\boxed{\text{For all } p,q \in P, \text{ count points in } P \text{ ~~on or~~ below } \overleftrightarrow{pq}}$

$\downarrow$ duality!

$\boxed{\begin{array}{l}\text{Given } n \text{ lines } P^* \\ \quad \text{For all } p^*, q^* \in P^* \\ \quad\quad \text{count lines in } P^* \text{ above } p^* \wedge q^*.\end{array}}$

level of a vertex = # lines below it



Once we compute arrgh(P^*)

we can compute level of every vertex and edge in $O(1)$ time each

$\boxed{O(n^2) \text{ time}}$

Ham-Sandwich Theorem

Given two point sets B and R

Find a line that bisects both.

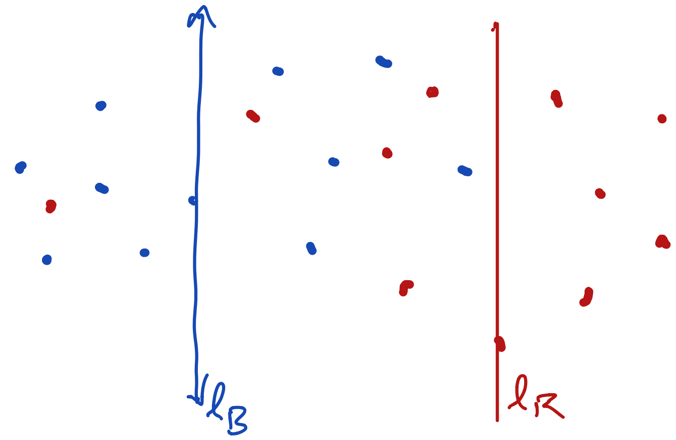
3D: Bread + ham + bread

There is a plane that bisects all three

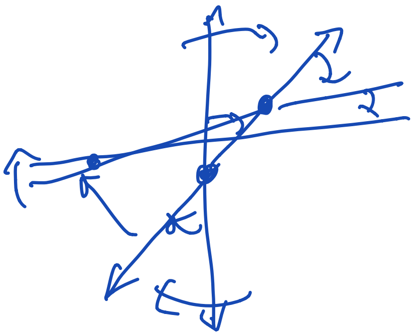
bread + Chicago + moon

Wlog $|R|$ and $|B|$ are odd

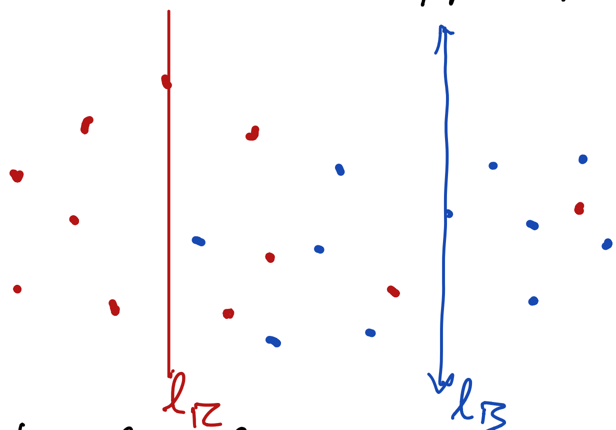
Let ℓ_B and ℓ_R be vertical bisectors (median x-coords)



Rotate plane continously
maintain ℓ_B and ℓ_R
change continuously



WLOG ℓ_B is initially left of ℓ_R
after half turn, swapped places



So at some angle, $\ell_R = \ell_B$

Duality: Given two sets of lines R^* and B^*

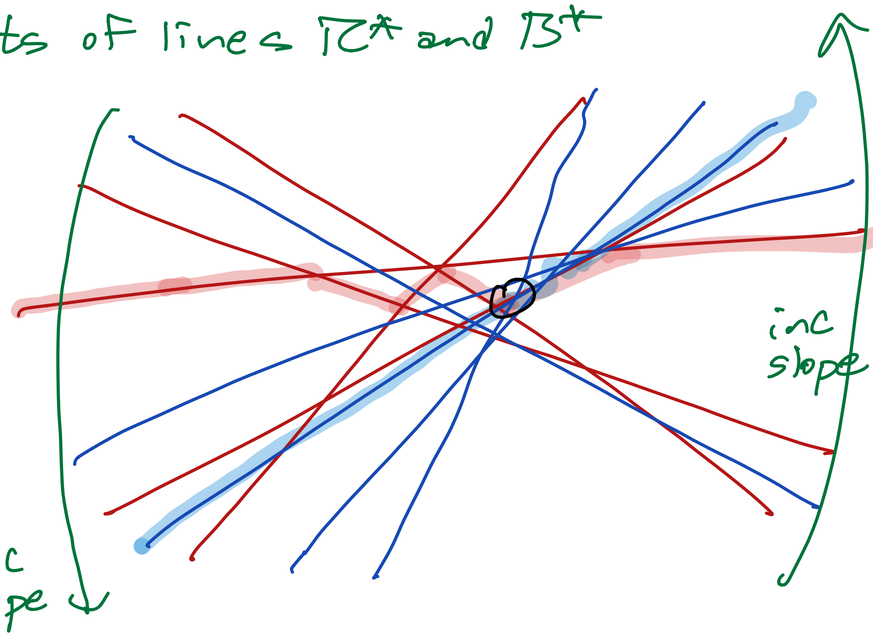
we want a point
with half of each set above

$O(n^2)$

Build both arrghs

Walk along median levels

Continuity $\Rightarrow$
median levels
intersect



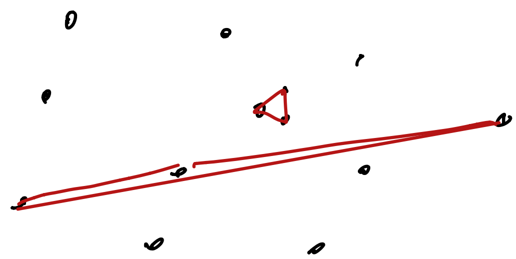
Any intersection point is dual of a ham-sandwich line

$O(n^2)$ + complexity of median red level
+ complexity of median blue level $\Big\}$ $\boxed{O(n^2)}$

We don't know this! $O(n^{4/3})$ but $n2^{\Omega(\sqrt{\log n})}$

$O(n \log n)$ time algorithm exists

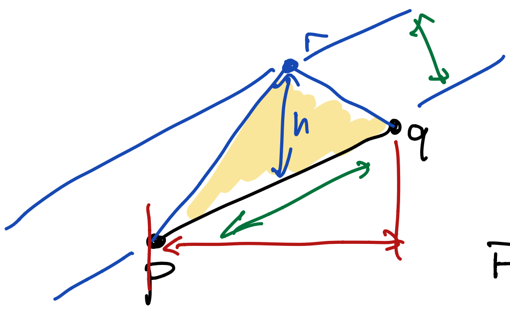
Minimum-area triangle



Naive: $O(n^3)$ time

$\downarrow$

$O(n^2)$ time as follows



$\text{Area}(\Delta pqr) = \frac{1}{2} b h$

where $\boxed{b = |p.x - q.x|}$

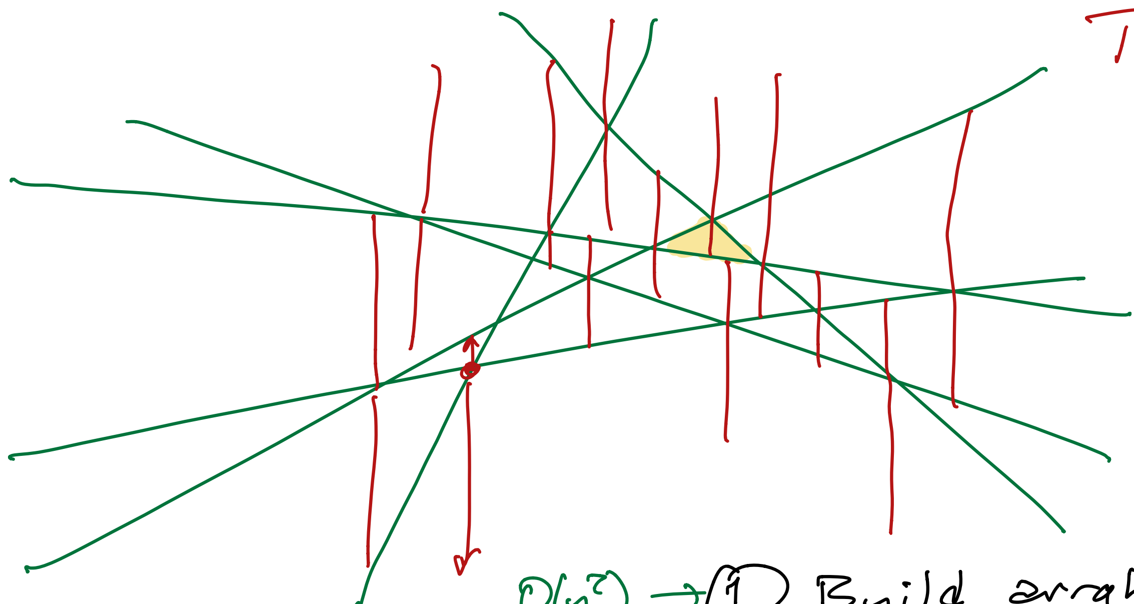
$h =$ vert dist from r to $\overleftrightarrow{pq}$

For each p and q,
we want point r closest to $\overleftrightarrow{pq}$

dual

$\boxed{\begin{array}{l}\text{Given lines } P^* \\ \quad \text{for each } p^* \text{ and } q^* \\ \quad\quad \text{we want line } r^* \text{ closest} \\ \quad\quad\quad \text{above or below } p^* \cap q^*\end{array}}$

Trapezoidal Decomposition!



$O(n^2) \rightarrow$ (1) Build arrgh(P^*)

$O(n^2) \rightarrow$ (2) Build trap. decomp.

Two candidates r^+ and r^-
for each p and q $\Rightarrow O(n^2)$

Can we do better? Nobody knows!

3SUM-hard problems

3SUM: Given a set X of n numbers
are there elements $a, b, c \in X$
s.t. $a+b+c=0$?

$O(n^2)$ time easy exersise

Define $\hat{X} = \{(x, x^3) \mid x \in X\}$

Claim: $\hat{X}$ has 3 collinear points iff X has 3 elements $\Sigma = 0$

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = \underline{(a+b+c)}(b-a)(c-a)(c-b) \qquad \square$$

Best 3SUM: Chan 2018: $O(n^2 \log^c \log n / \log^2 n)$ time

Best 3-collinear: $O(n^2)$

3SUM conjecture: $O(n^{1.99999\ldots9})$ time is impossible