CS 498 TC ✧ Computational Geometry ✧ Spring 2022

Homework 4

Due Friday, April 29, 2022 at 8pm

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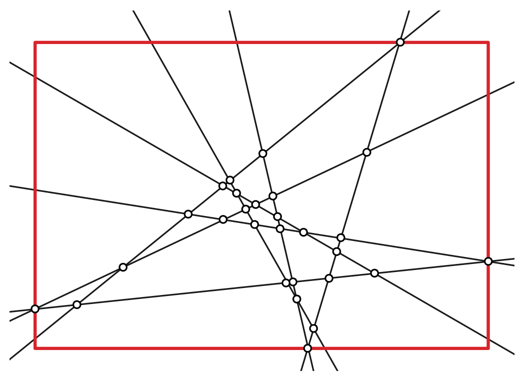
1. Suppose you are given a set L of n lines in the plane in general position. Describe an algorithm to compute the smallest axis-aligned rectangle that contains all $\binom{n}{2}$ vertices of the arrangement of L. For full credit, your algorithm should run in $O(n \log n)$ time. *[Hint: What is the dual of the rightmost arrangement vertex?]*



2. Suppose you are given a set S of n line segments in the plane.

   (a) Describe and analyze an algorithm that either finds a line that intersects every segment in S, or correctly reports that no such line exists. *[Hint: What is the dual of a line segment?]*

   (b) Describe an analyze a *faster* algorithm for the special case where each segment in S is either horizontal or vertical.

3. In class we saw the classical *funnel* algorithm to compute shortest paths inside a triangulated simple polygons. How would you modify this algorithm to find shortest paths in a polygon with holes?

   (a) Describe and analyze an algorithm to compute the shortest path between two given points in the interior of a polygon with *one* hole. *[Hint: Which way does the path go around the hole?]*

   (b) Describe and analyze an algorithm to compute the shortest path between two given points in the interior of a polygon with *two* holes.

   (c) **[Extra credit]** Describe and analyze an algorithm to compute shortest paths in a polygon with h holes; analyze your algorithm as a function of both n (the total number of polygon vertices) and h (the number of holes).

   In all cases, you can assume that you are given a triangulation of the input polygon. For full credit, your algorithms should run in $O(n)$ time (for any constant h).

4. A set $\mathcal{X}$ of objects in the plane is called *a family of pseudodisks* if (1) the boundary of each object in $\mathcal{X}$ is a simple closed curve, (2) the boundaries of any two objects in $\mathcal{X}$ either cross each other at exactly two points or do not intersect at all.

   A seminal result of Kedem, Livne, Pach, and Sharir,[1] which arises in the analysis of Minkowski sums, states that the union of any family of n pseudodisks consists of at most $O(n)$ boundary segments. This problem asks you to prove two special cases of this result.

   (a) Let $\mathcal{C}$ be a collection of n circular disks in the plane in general position, meaning no two disks are tangent. Prove that the boundary of the union of $\mathcal{C}$ consists of at most $O(n)$ circular arcs or complete circles. (Any set of circular disks in general position is clearly a family of pseudodisks.)

   (b) Let $\mathcal{X}$ be a family of n *convex* pseudodisks *that all contain a common point in their interiors*. Prove that the boundary of the union of $\mathcal{X}$ consists of at most $O(n)$ boundary segments. *[Hint: Convexity isn't actually necessary, but it simplifies the proof. First show that the union of $\mathcal{X}$ is star-shaped.]*

[1] Klara Kedem, Ron Livne, János Pach, and Micha Sharir. On the union of Jordan regions and collision-free translational motion amidst polygonal obstacles. *Discrete Comput. Geom.* 1(1):59–71, 1986.