Computational Geometry usually low-dimensional

Algo + DS for geometric problems
input/output/intermediate data
= points, lines, circles, boxes...

Post-office problem [knuth] Where is nearest P.O.?

Std solution:
Partition the plane into regions closest to each P.O.
Represent in a data structure supports queries.

Shortest paths, avoiding obstacles

Given polygonal obstacles $P_1 \ldots P_k$

Intermediate data structures?
Prove structure?
**Convex hulls**

Given $n$ points in the plane $P$, find the smallest convex polygon containing $P$.

If $X$ is convex, then $p \in X$ and $q \in X$, then $pq \subseteq X$.

Let $P = \{p_1, p_2, \ldots, p_n\}$, then

$$\text{conv}(P) = \left\{ \sum_{i=1}^{n} \lambda_i p_i \mid \sum_{i=1}^{n} \lambda_i = 1 \text{ and } \lambda_i \geq 0 \text{ for all } i \right\}$$

The smallest inclusion-minimal

For every convex set $X$, if $p \in X$ then $\text{conv}(P) \subseteq X$.

$\text{conv}(P)$ = intersection of all convex sets containing $P$.
n=1

n=2

n=3

[1 2 3]

CCW

[1 3 2]

CW

Assume \( P_2 \) is leftmost

Which line has bigger slope?

\[
\frac{y_2 - y_1}{x_2 - x_1} < \frac{y_3 - y_1}{x_3 - x_1}
\]

\[(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1) > 0 \quad \text{ccw} \]

\[
\text{det} \begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{bmatrix} > 0
\]

\[
\text{det} \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} > 0
\]

2d analogue of comparisons \( a > b \)? \( |\frac{a}{b}| < 0 ? \)

Degeneracies Numerical errors
**Jarvis’ March** ("selection hull")

1. Find leftmost point $P_0$
   - $n-1$ x-coord comparisons

2. Repeat
   - Find next hull vertex
   - $n-2$ orientation tests until we reach $P_0$ again

Running time $O(nh) = O(n^2)$

```plaintext
next = 1
for i = 2 to n
  if next = curr or Orient(P[curr], P[next], P[i]) < 0
    next = i
```