COVID stuff Admin stuff

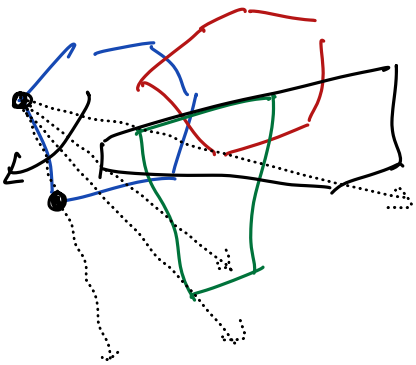
Last time: Convex hulls
- Jarvis march $O(nh) \leq O(n^2)$
- Graham's scan $O(n \log n) = \text{sort} + O(n)$
- Chan's algorithm "shatter hull" $O(n \log h)$ optimal!

Little bird tells us h.

- Split input P into $O(\frac{n}{h})$ subsets of size $O(h)$
- Compute convex hull of each subset

$$O(\tfrac{n}{h}) \times O(h \log h) \quad \text{[Graham]}$$
$$= O(n \log h)$$

- Wrap a string around these smaller convex hulls



$O(h)$ Repeat h times:

$O(\frac{n}{h}) \times O(\log h)$ → Find tangent rays from prev output vertex to all $O(\frac{n}{h})$ subhulls

↑ "binary" search

→ Choose next output vertex

$O(n \log h)$ altogether

Implement bird by exponential search

Try $h = 3$ then $h = 9, 81, \cdots 3^{2^k} = h_k$

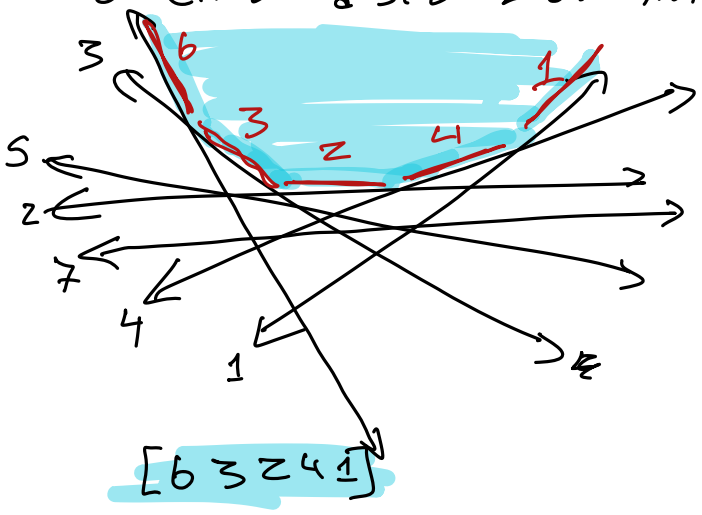
$$O(n \underbrace{\log h_0}_{\lg 3} + n \underbrace{\log h_1}_{2 \lg 3} + n \underbrace{\log h_2}_{2^2 \lg 3} + \cdots + n \underbrace{\log h^2}_{2^k \lg 3})$$

geom. series → dominated by last term

$$\leq O(n \cdot \log(h^2))$$
$$= O(n \log h)$$

Halfspace Intersection

Given a set L of lines in plane $y = ax - b$



Output description of upper envelope of L

Array of indices in order

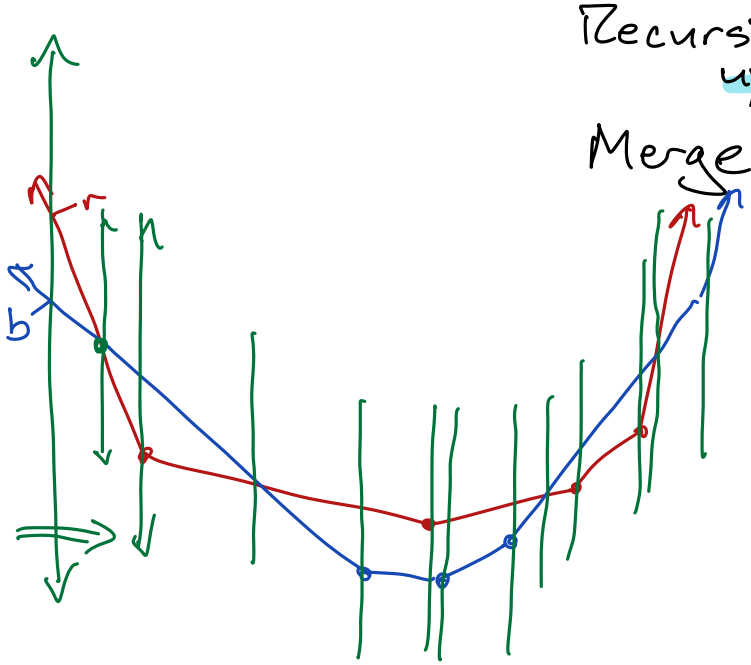
Vertex coords: solve system
$y = a_i x - b_i$
$y = a_j x - b_j$ for (x, y)

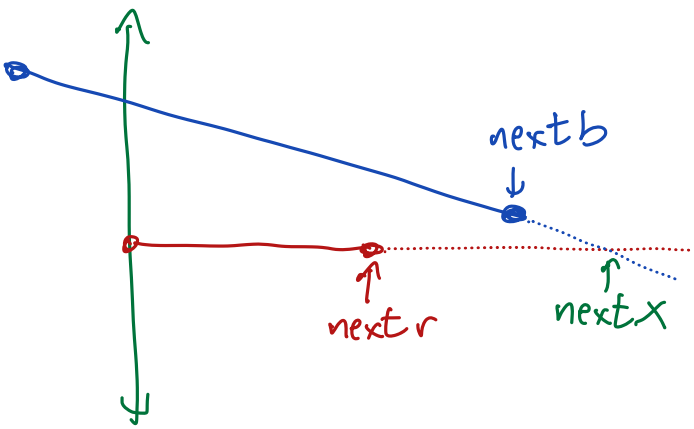
"Merge-envelope"

Split lines L into two subsets $B + R$

Recursively compute upper(B) and upper(R)

Merge → sweepline algorithm





Intuition:
Vertical line sweeps ⟹ maintains intersection with upper(B) and upper(R)

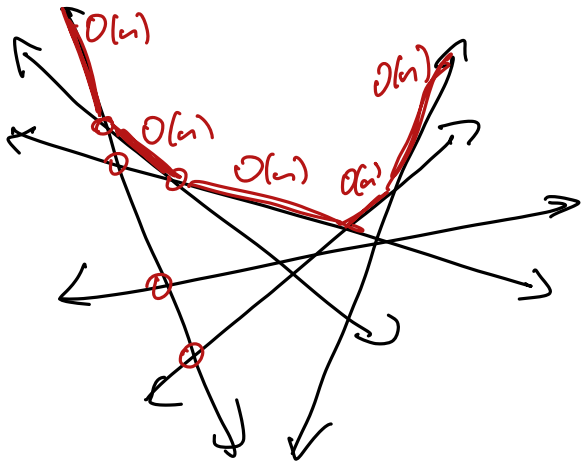
Actually:
Stop only at events
- right end of red edge
- right end of blue edge
- intersection of current red + blue lines

$O(n)$ time

At each event, compute next event in $O(1)$ time

Summary: $T(n) = 2T(\frac{n}{2}) + O(n) = O(n \log n)$

Output-sensitive



leftmost edge of envelope
= lowest slope

In $O(n)$ time we can find the next line on the upper envelope

$O(nh)$ time
where h = # edges on upper envelope

Convex hull problem
n points

upper envelope
n lines

reality
2n numbers

Projective duality

coordinates
$p = (a, b)$

$\longleftarrow (a, b) \longrightarrow$

p^* $y = ax - b$

ℓ: $y = ax - b$

$\ell^* = (a, b)$

$p \in \ell$

Incidence

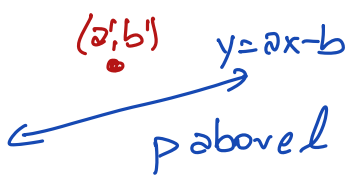
$b' = a \cdot a' - b$

$a \cdot a' = b + b'$

$\ell^* \in p^*$

$\ell = \overleftrightarrow{pq}$

$\ell^* = p^* \cap q^*$



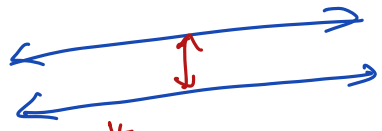
Order

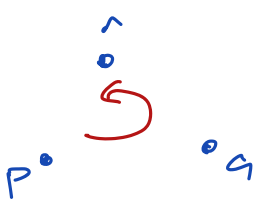
$b' > a \cdot a' - b$

$a \cdot a' < b + b'$

ℓ^* above p^*

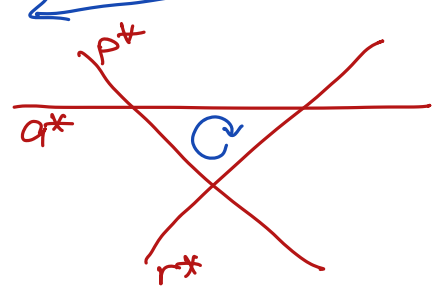
vertical dist.



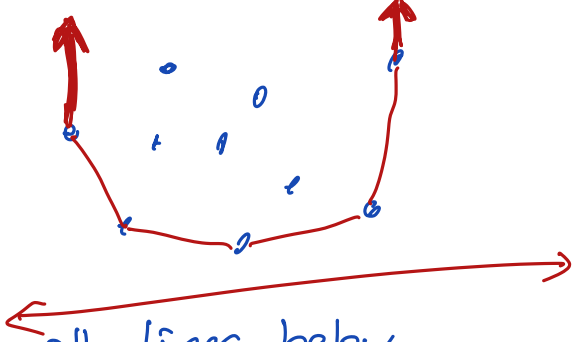


Orientation

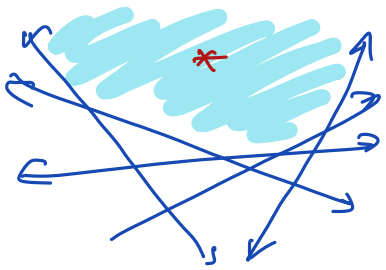
$$\begin{vmatrix} 1 & p_1 & p_2 \\ 1 & q_1 & q_2 \\ 1 & r_1 & r_2 \end{vmatrix} > 0$$



Convex Stuff



all lines below every point in P
= lower convex hull
vertices from left to right



upper envelope
all pts above every line in L
= upper envelope edges by increasing slope

DUALITY IS A TYPE-CAST
not a "transformation"