**Projective Duality**

**Intuition:** Data are points.

Under the hood: pairs of numbers

But pairs of numbers can represent other objects

\[
(\alpha, \beta) \quad (\delta, \gamma)
\]

Ex: \[
\exists x | \alpha x \leq \beta \\
[\alpha, \beta]
\]

\[
\delta x + \gamma y = 1
\]

---

**Halfplane Intersection/Upper Envelope**

Given a set of lines in \( \mathbb{R}^2 \)

\[
y = ax - b
\]

Compute representation of upper envelope of \( L \)

Sequence of supporting lines in order from left to right

**Convex**

We can compute vertices by solving linear system

\[
\begin{align*}
    y &= \delta_6 x - b_6 \\
    y &= \delta_2 x - b_2
\end{align*}
\]

---

**Divide and Conquer** → merge sort

- Partition \( L \) into \( R, U, B \) disjoint equal size
- Recursive compute upper envelopes \( \hat{E} \) and \( \hat{B} \)
- Merge
Merge using a sweep algorithm
Sweep vertical line \( \ell \) from left to right
Record intersections with \( \hat{I} \) and \( \hat{B} \)
- index \( r \) of edge of \( \hat{I} \)
- index \( b \) of edge of \( \hat{B} \)
- which point is higher
This info will change at
- next red vertex
- next blue vertex
- next red-blue intersection whichever is next

Init: \( r = 1 \), \( b = 1 \), higher = lower slope
\( \text{curx} = -\infty \)
Repeat:
\( \text{curx} \leftarrow \text{next event} \)
if higher intersection changes
record new higher edge
until \( \text{curx} = \infty \)

\( T(n) = 2T(\frac{n}{2}) + O(n) \Rightarrow O(n \log n) \) time

Output-sensitive
Start with line \( \ell \) with min slope
Repeat
find next vertex \( \leftarrow O(n) \) time
update \( \ell \)
until \( \ell \) has max slope

\# iterations = \# vertices of upper envelope
\( O(nh) \) time

This is Janis march.
**Duality**

**Primal**
- point $p = (a, b)$
- line: $y = a'x - b'$

**Dual**
- line $p^* : y = bx - a'$
- point $l^* = (a', b')$

**Math**
- $(a, b)^T$
- $(a', b')$

$aa' = b + b'$

**Incidence:**
- $p \in l$
- $l = P_1P_2$

**Order:**
- $P_1$ left of $P_2$
- $P_1$ above $P_2$

**Vertical Distance**
- $P$ above $l$
- $P$ below $l$

**Orientation**
- $P_1P_2P_3$ ccw

**Convex stuff**
- lower convex hull
  - all lines below every pt in $P$
  - vertices of lower hull left to right
- upper envelope
  - all points above all lines in $L$
  - edges of upper envelope left to right
  - vertices