HW1 solutions out
HW2 out \(\rightarrow \) due next Thu
Midterm due following Tuesday 8pm

Last time: DCEL – data structure for planar maps

\(V - E + F = 2\) planar iff

planar maps
plane graphs
planar embeddings
planar straight-line graphs

\[\text{prev}(d) \rightarrow \text{left}(d) \rightarrow \text{innext}(d) \rightarrow \text{hnext}(d) \rightarrow \text{head}(d) \rightarrow \text{rev}(d) \rightarrow \text{right}(d) \rightarrow \text{hprev}(d) \rightarrow \text{tail}(d)\]

left\((d^*)\) = head\((d^*)\)
right\((d^*)\) = tail\((d^*)\)
head\((d)\) = left\((d^+)\)
tail\((d)\) = right\((d^+)\)

delete
contract

\(\rightarrow\)
Euler's Formula: connected plane graph 
\[ V - E + F = 2 \]

Base case: \( V = 1 \) \( E = 0 \) \( F = 1 \)

Otherwise: \( E \geq 1 \)
- Pick any edge \( e \)
  - Case 1: \( e \) is not a bridge 
    - delete \( e \)
    \[ E \rightarrow F \rightarrow V \rightarrow \checkmark \]
  - Case 2: \( e \) is not a loop 
    - contract \( e \)
    \[ E \rightarrow V \rightarrow \checkmark \]

All deleted edges \( \rightarrow \) dual spanning tree

All contracted edges \( \rightarrow \) spanning tree!

Trapezoidal decompositions in \( O(n \log n) \) time \( \checkmark \)

How do we support point location?

- Don't sweep
- Randomized Incremental Construction
  - Insert segments in random order
  - Update decomposition after each insertion.
Inserting a new segment pq into an existing decomposition:
- Locate p, split its trap \(0(1)\) each
- Walk along pq, splitting traps
- Reach q, split its trap \(0(1)\)
- Merges \(0(1)\) each

Define \(\text{deg}(pq) = \# \text{traps in new decomposition that touch } pq\)

\[ \Rightarrow O(\text{deg}(pq)) + \text{pt. loc. time} \]

\[= O(4) \text{ time } \quad \text{in worst case} \]

Updating a DCEL is inefficient (unless we are clever)

So don't do that.

**Lightweight data structure**

- Store set of trapezoids
- Each trap stores \(\leq 4\) segments defining it
- \(\leq 4\) neighboring traps thru vertical walls
Backward analysis—imagine running randomized incremental algo backward randomized decremental algorithm

Choose a random segment
Delete it from trap decompo ← Expected time
Recurse

Total expected time = $O(n)$

What is $E[\text{deg}(s)]$ where $s$ is chosen randomly?

$$
\frac{1}{n} \cdot \sum_{i=1}^{N} \text{deg}(s_i) = \frac{1}{n} \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} [s_i \text{ touches } \Delta_j]_{\text{trapezoid}}
$$

$$
= \frac{1}{n} \cdot \sum_{j=1}^{N} \sum_{i=1}^{N} [s_i \text{ touches } \Delta_j]_{\text{trapezoid}}
$$

Euler ⇒ $N=O(n)$

$$
\leq \frac{1}{n} \cdot 4N = O(1)
$$