Write team names on HW submissions.

Date structures properties of planar maps.

Planar straight-line graph

- Vertices = points in \( \mathbb{R}^2 \)
- Edges = segments
- no crossings
- Faces = components of \( \mathbb{R}^2 \setminus VUE \)

Planar graph \((V,E)\) abstract

Planar map \((V,E,F)\) topological

\[ \rightarrow \text{Planar straight-line graph} \quad \text{geometry} \]

Incidences adjacency list

Adjacency list

doubly-connected circular list of neighbors incident edges

Split every edge into two directed “half-edges”/darts

\[ \text{tail}(h) \quad \text{head}(h) \quad \text{head}(\text{rev}(h)) = \text{tail}(\text{rev}(h)) \]
PSLG: Vertices have coordinates
\[ \begin{array}{c|c|c} x & y & \text{First} \\ \hline \text{Coords.} & \text{some halfedge } h & \text{where } \text{vehead} (h) \end{array} \]

Half-edges "store" lots of ptrs

Half-edges store lots of ptrs.

Faces!
Every halfedge has left share right share

\[
\begin{align*}
\text{lnext}(h) &= \text{rev}(\text{hnext}(h)) \\
\text{tnext}(h) &= \text{rev}(\text{hnext}(\text{rev}(h)))
\end{align*}
\]

Vertices, halfedges, faces store auxiliary information.

In particular every face F stores one halfedge h where left(h) = F

If G is disconnected, every face stores a list of darts, one on each boundary cycle.
Dual map $G^*$ (G connected)

Vertices $F^*$

Edges $E^*$

Faces $V^*$

Data structure for $G$

Is a data structure for $G^*$

$V - E + F = 2$

$E \leq 3V - 6$