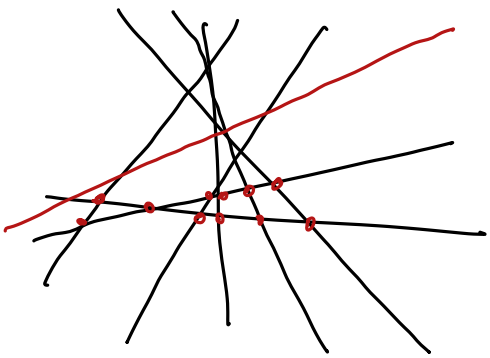
What should I do at end of semester?

Paper presentations (last week April)
Registration form – due April 15 ish
↑
end of classes Jeff traveling 1st week of May

May 11 – exam

Line Arrangements

Given set L of n lines in the plane
build their arrangement



Sweep: $O(n \log n + k \log n)$ [GP]
$= O(n^2 \log n)$ $k = \binom{n}{2}$

Randomized Incremental:
$O(n \log n + k) = O(n^2)$

Incremental: $O(n^2)$ time simple

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general position: $\#\text{vertices} = \binom{n}{2}$

$\#\text{edges} = n^2$

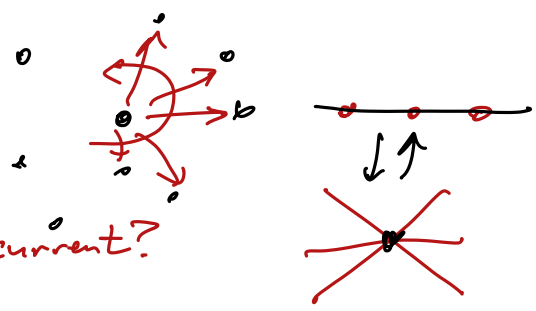
$\boxed{V - E + F = 1}$ → $\#\text{faces} = \binom{n}{2} + n + 1$ ← $\boxed{F(n) = \begin{cases} 1 & \text{if } n = 0 \\ F(n-1) + n \end{cases}}$

Motivation:

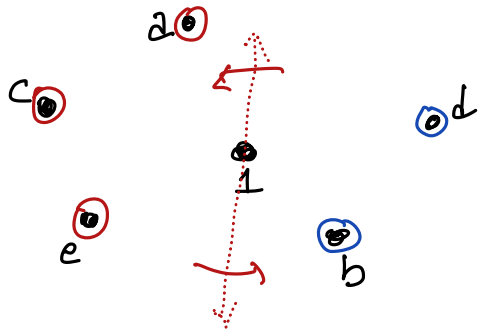
via duality

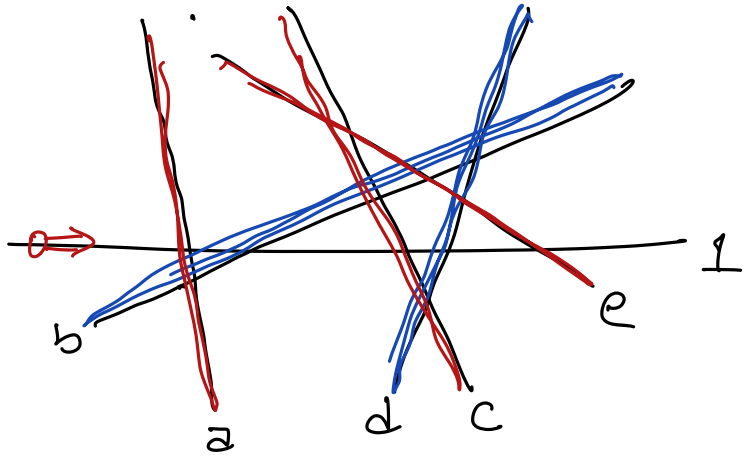
- Given n points, are any 3 collinear?
  - $O(n^3)$ brute force
  - ⟹ $O(n^2 \log n)$ time via sorting
  - $O(n^2)$ duality

    Given n lines, are any 3 concurrent?



- Find cyclic orders around each point





(Halfplane) Discrepancy:

Antialiasing

Let P be set of points in $[0,1]^2 = \square$

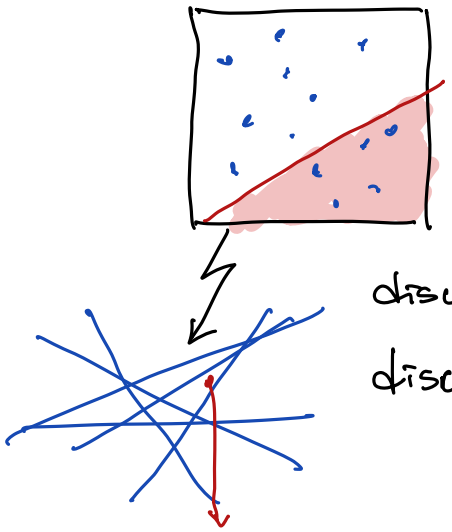
For any halfplane h, define

$\mu(h)$ = area of $h \cap \square$

$\mu_P(h) = |P \cap h| / |P|$

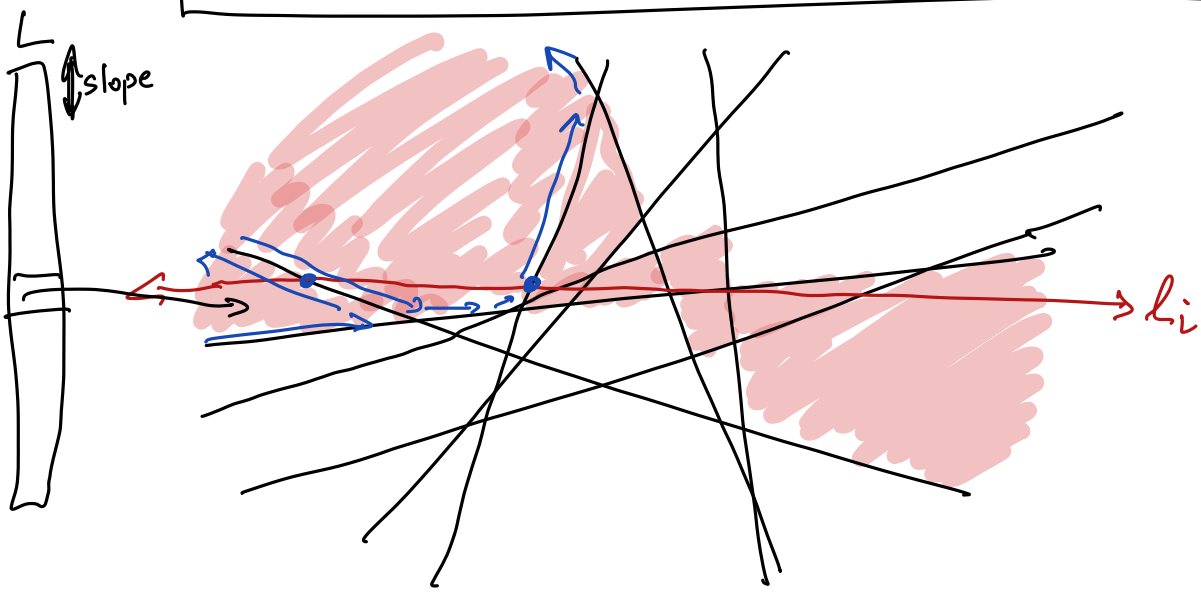
discrepancy $\Delta_P(h) = |\mu(h) - \mu_P(h)|$

discrepancy of P = $\sup_h \Delta_P(h)$



Incremental algorithm:

$\boxed{\text{For } i \leftarrow 1 \text{ to } n \quad \text{insert } \ell_i \text{ into the arrgh by walking thru } \underline{\text{zone of } \ell_i}}$



Zone(ℓ, L) = set of cells in arrgh(L) that intersect ℓ

① Find unbounded cell containing "left end" of ℓ

binary search by slope

② repeat

walk around boundary of current cell to find next cell intersecting ℓ_i

Insertion time $= O(\log n) + O(\text{complexity of zone}(\ell, L))$

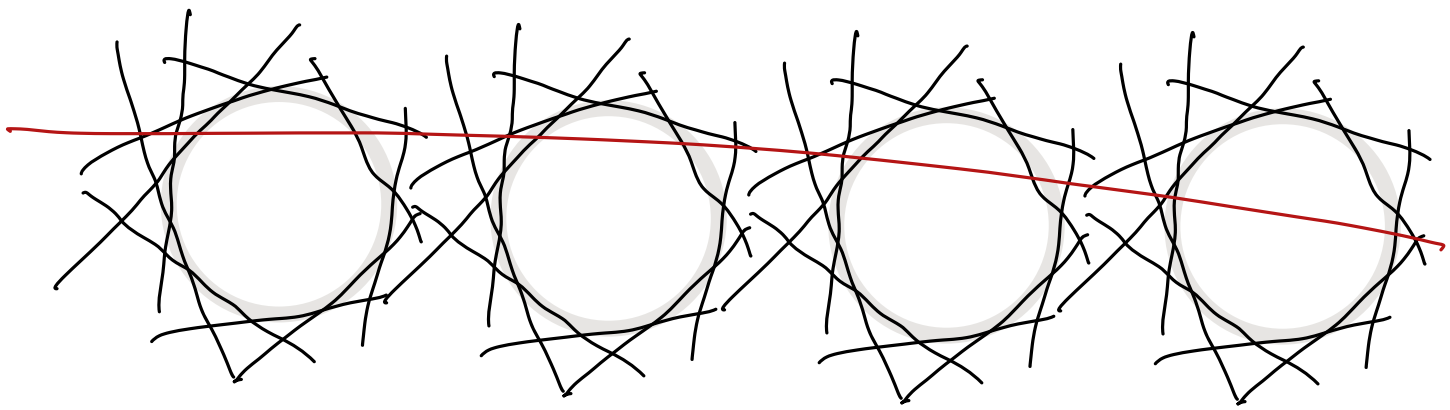
$\sum_{\text{cell}} \#\text{edges of cell}$

Zone Theorem:

$$\boxed{\#\text{Zone}(\ell, L) \le 6 \cdot |L|}$$

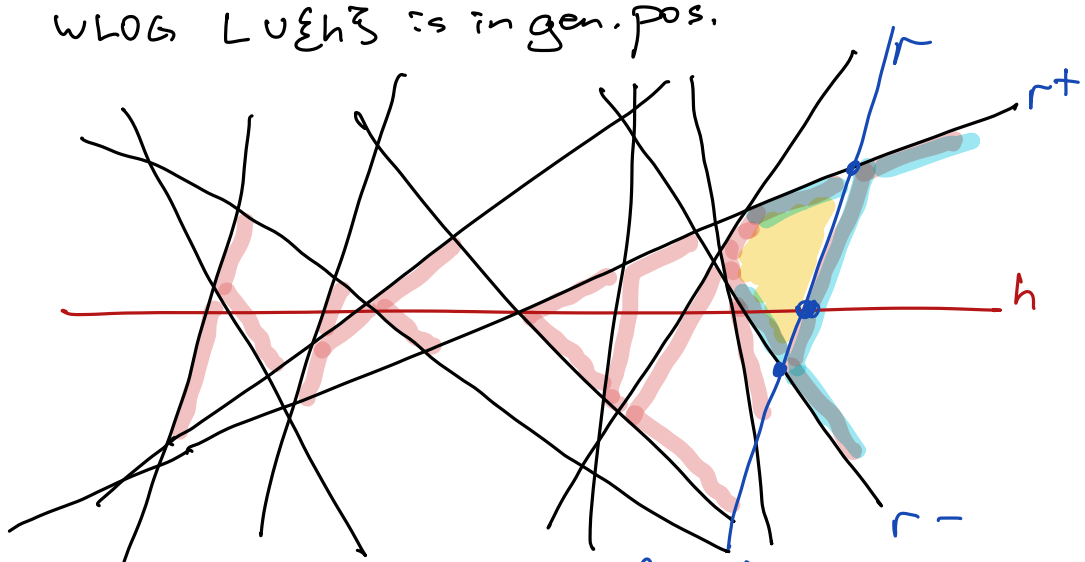
Intuition: Average #edges in a random face ≤ 4

Counterintuition: But one face can have n edges



Proof: Given set L and a horizontal line h

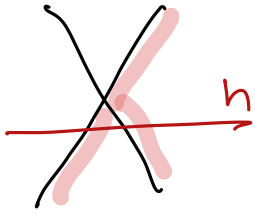
WLOG $L \cup \{h\}$ is in gen. pos.



Count right edges in zone(h, L)

Claim: # right zone$(h, L) \le 3n-3$ unless $n \le 1$

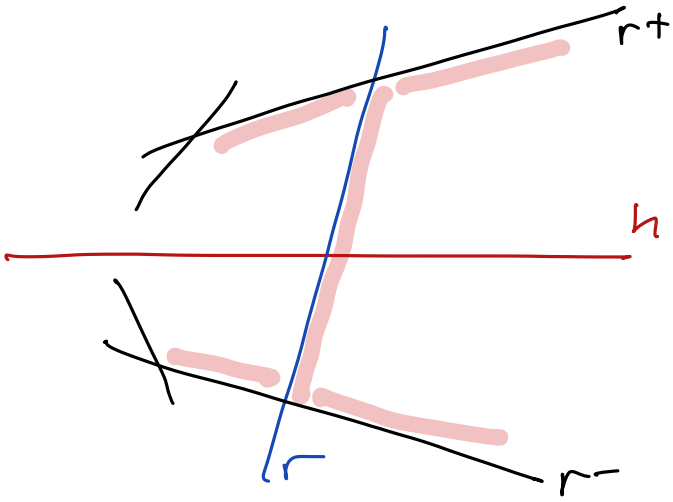
Base case: $n = 2$



right edges $= 3 = 3 \cdot 2 - 3$ ✓

When $n > 2$:

r = line intersects h furthest right



Let r^+ and r^- be lines intersecting r just above + below h

Deleting r removes at most 3 right zone edges

- right edge on r vanishes
- two right edges meeting at $r \cap r^+$ merge
- ... $r \cap r^-$

IH $\Rightarrow$ zone$(h, L-r)$ has $\le 3n-6$ right edges

$\square$

Symmetrically zone(h, L) has $\le 3n-3$ left edges

$\Longrightarrow \le 6n-6$ edges $\square$

Really $\le \frac{9}{2} n - O(1)$