What should I do at end of semester?

Paper presentations last week April
Registration form due April 15ish
end of classes Jeff trading 1st week of May
May 11 exam

Line Arrangements
Given set \( L \) of \( n \) lines in the plane, build their arrangement.

Sweep: \( O(n \log n + k \log n) \) \([GP]\)
\( = O(n^2 \log n) \)
\( k = \binom{n}{2} \)

Randomized Incremental:
\( O(n \log n + k) = O(n^2) \)

Incremental: \( O(n^2) \) time simple

general position:
# vertices = \( \binom{n}{2} \)
# edges = \( n^2 \)
\( \leq V - E + F = 1 \)
# faces = \( \binom{n}{2} + n + 1 \)

Motivation:
via duality

- Given \( n \) points, are any 3 collinear?
  \(-O(n^3)\) brute force
  \( \Rightarrow -O(n^2 \log n) \) time via sorting
  \(-O(n^2)\) duality
  Given \( n \) lines, are any 3 concurrent?
- Find cyclic orders around each point
(Halfplane) Discrepancy:

Let $P$ be the set of points in $[0,1]^2 = \square$.

For any halfplane $h_i$, define

$$\mu(h) = \text{area of } h \cap \square$$

$$M_P(h) = \frac{|P \cap h|}{|P|}$$

Discrepancy $\Delta_P(h) = |\mu(h) - M_P(h)|$

Discrepancy of $P = \sup \Delta_P(h)$

Incremental algorithm:

For $i = 1$ to $n$:
- Insert $l_i$ into the arch by walking through zone of $l_i$.

Zone $(l_i, L) = \text{set of cells in arch}(L) \text{ that intersect } l$
1. Find unbounded cell containing "left end" of $l$
   binary search by slope

2. repeat
   walk around boundary of current cell
   to find next cell intersecting $l_i$

Insertion time $= O(\log n) + O(\text{complexity of zone}(l,L))$

$$\sum_{\text{cell}} \#\text{edges of cell}$$

**Zone Theorem:**

$$\#\text{Zone}(l,L) \leq 6 \cdot L$$

Intuition:
Average \#edges in a random face $\leq 4$

Counter-intuition:
But one face can have $n$ edges

**Proof:**
Given set $L$ and a horizontal line $h$

WLOG, $L \cup h\mathbb{Z}$ is in gen. pos.

Count right edges in $\text{zone}(h,L)$
Claim: \( \# \text{right zone}(h, L) \leq 3n-3 \) unless \( n \leq 1 \)

Base case: \( n = 2 \)

\[
\#	ext{right edges} = 3 = 3 \cdot 2 - 3 \checkmark
\]

When \( n > 2 \):

\( r = \) line intersects \( h \) furthest right

Let \( r^+ \) and \( r^- \) be lines intersecting \( r \) just above/below \( h \)

Deleting \( r \) removes at most 3 right zone edges

- right edge on \( r \) vanishes
- two right edges meeting at \( r \) \( r^+ \) merge

\[\square\]

IH \( \Rightarrow \) zone\((h, L-r)\) has \( \leq 3n-6 \) right edges

\[\square\]

Symmetrically zone\((h, L)\) has \( \leq 3n-3 \) left edges

\[\Rightarrow \leq 6n-6 \text{ edges} \square\]

Really \( \leq \frac{9}{2} n - O(1) \)