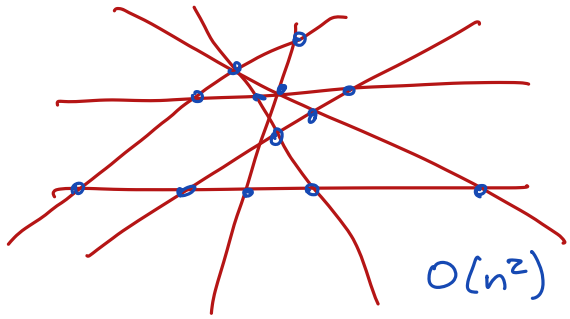
Paper presentations — last week April 20 min each

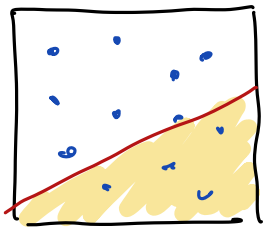
Register on Gradescope by April 15

Applications of Line Arrangements (via duality)



$O(n^2)$ time

(Half plane) Discrepancy



P – n points in $\square = [0,1]^2$

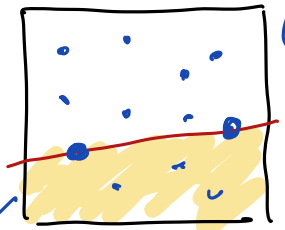
for any halfplane h define

$$\mu(h) = \text{area}(h \cap \square)$$

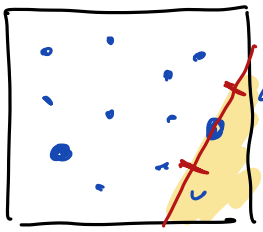
$$\mu_P(h) = |P \cap h| / n$$

discrepancy $\Delta(h) = |\mu(h) - \mu_P(h)|$

Given P compute $\longrightarrow$ $\Delta(P) = \max_h \Delta(h)$



$\binom{n}{2}$ lines of this form



$O(n)$ lines of this form ($O(1)$ per point)

$O(n^2)$ time by brute force

worst halfspace is either bounded by

line thru two points in P

or line thru one point in P at midpoint of $\ell \cap \square$

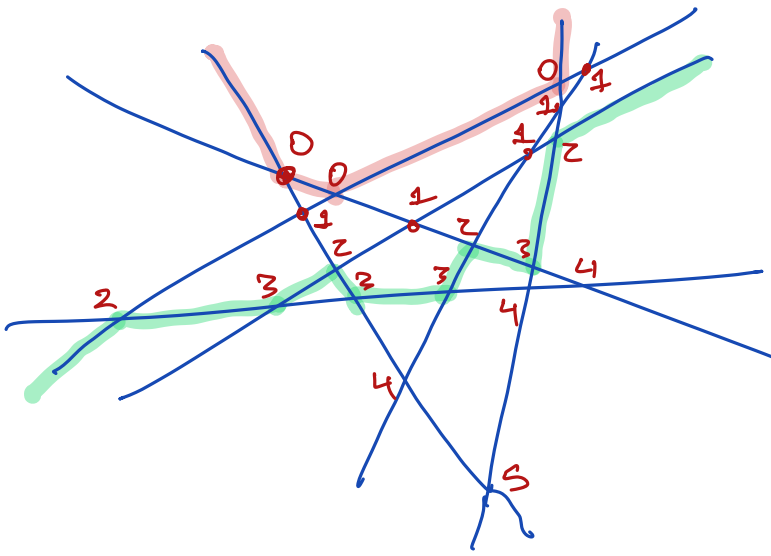
We can compute $\mu(h)$ for any h in $O(1)$ time

Bottleneck: compute $\mu_P(h)$ for all candidates h.

For every pair of points p, q, find # points below that line $\overleftrightarrow{pq}$

duality

For every pair of lines p^*, q^*, find # lines above point $p^* \cap q^*$



Build $\text{argh}(P^*)$

① Trace thru each level in $O(1)$ time per vertex

—or—

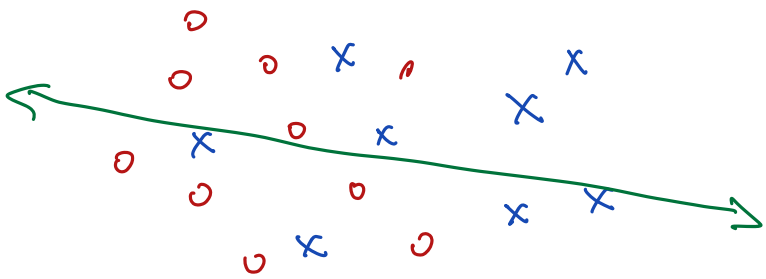
② Compute level of any vertex, WFS

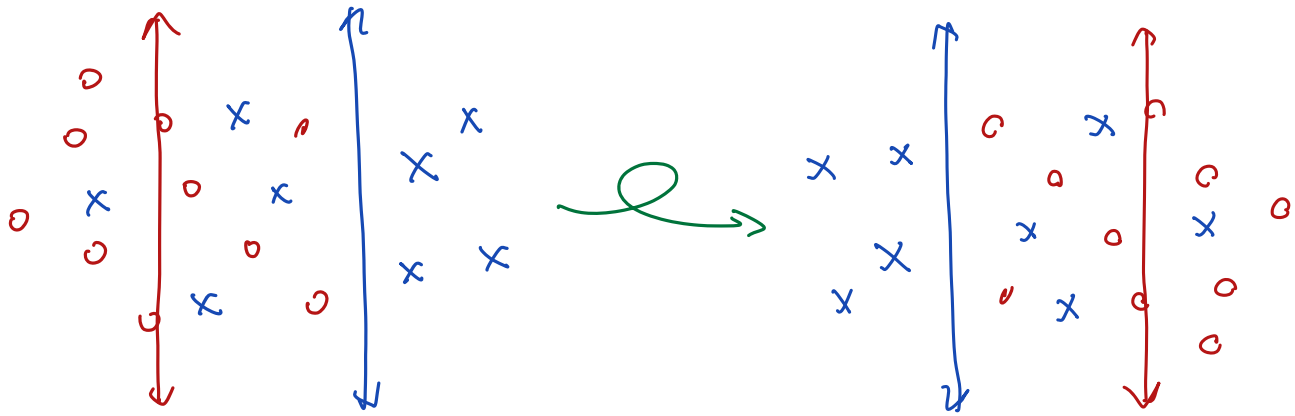
$\boxed{O(n^2) \text{ time}}$

$\boxed{\text{FASTER?}}$

Ham Sandwich Cuts

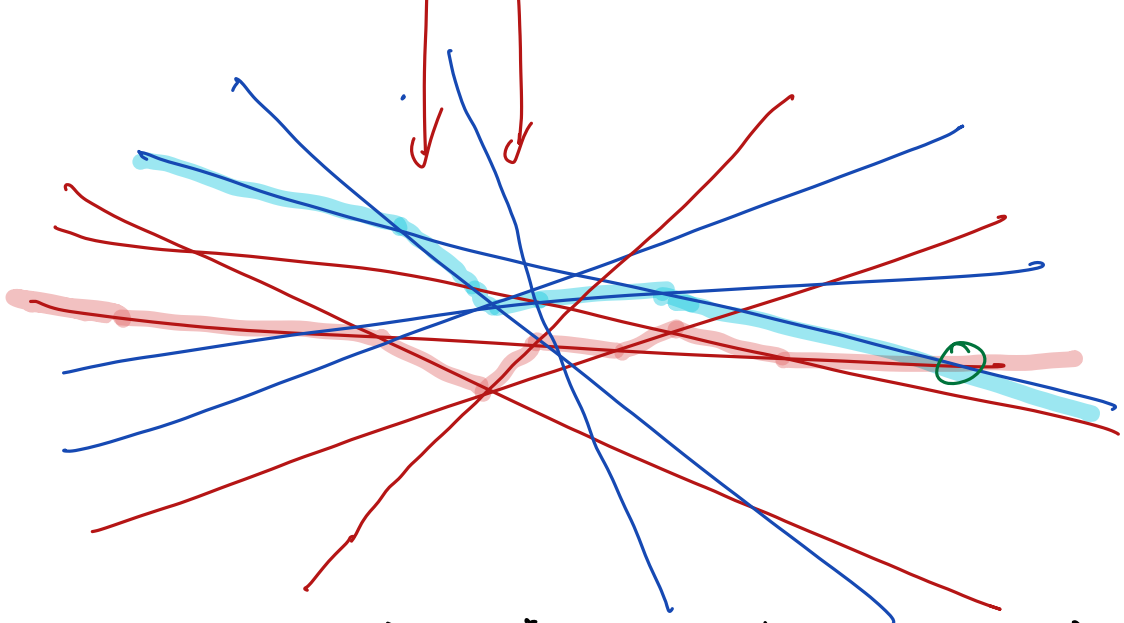
Given two point sets R and B find a line that bisects both sets





If we rotate points, maintain vertical bisectors, IVT → at some point lines coincide

duality



Build arrangements of R^* and B^* $\leftarrow O(n^2)$

Trace median levels in both arrgns $\leftarrow \boxed{O(n^2)}$ $O(n^{4/3})$

IVT $\rightarrow$ median levels intersect

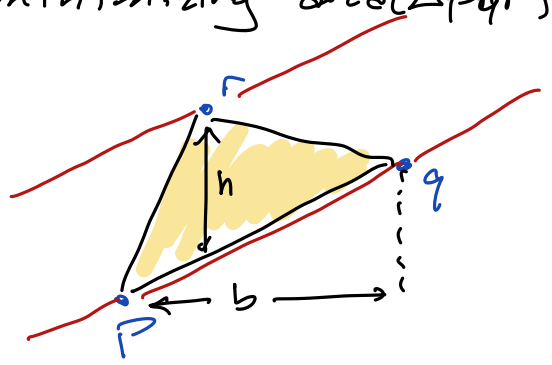
We don't know worst case complexity of median level!

$n \cdot 2^{\Omega(\sqrt{\log n})}$ $O(n^{4/3})$

<u>Minimum area triangle</u>

Given set P Find $p, q, r \in P$ minimizing area(Δpqr)

Brute force: $O(n^3)$



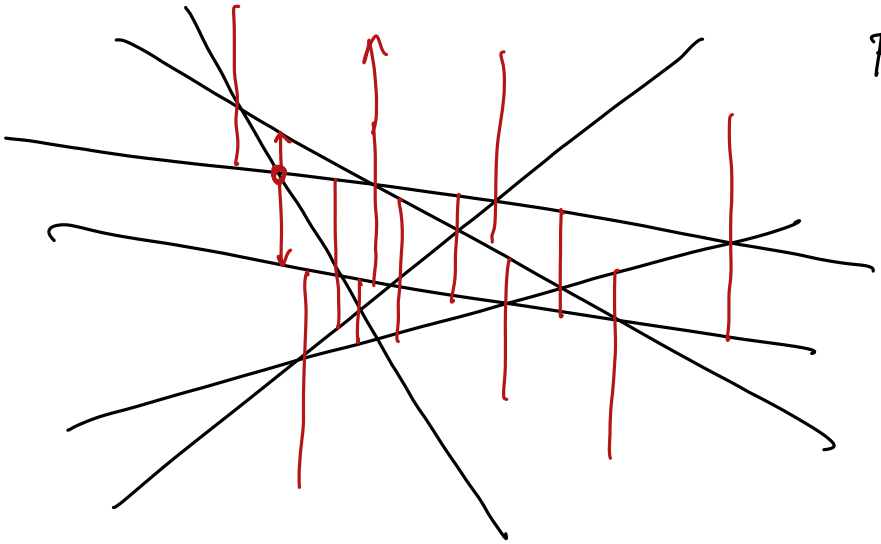
Area $= \frac{1}{2} b h$

Fix two points p and q

Min area $\Delta pqr \Leftrightarrow$

dist$(r, \overleftrightarrow{pq})$ is minimized

dualize



For each vertex in arrgh(P*)

we want closest line above or below

We can build arrgh + trap decomp in $O(n^2)$ time

Do any three points lie on a line?

Do any three lines pass thru a common point?

<u>3SUM</u>: Given a set X of n numbers do any three elements of X sum to 0?

$O(n^2)$ time

Define $\hat{X} = \{(x, x^3) \mid x \in X\}$

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(a-c)(b-c)(a+b+c)$$

Matching $\Omega(n^2)$ lower bound in <u>limited</u> model

Grønlund Pettie 2014: $O\left(n^2 \frac{\log^2 \log n}{\log n}\right)$

⋮

Chan 2018 $O\left(n^2 \frac{\log^c \log n}{\log^2 n}\right)$ ← Fastest known

<u>3SUM conjecture</u>: $O(n^{2-\delta})$ time is impossible for all $\delta > 0$ even for integers between $-n^3$ and n^3.