

Project Proposal CS598JGE, Fall 2009: Partitioning quadrilateral meshes into structured submeshes

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Quadrilateral meshes have many important applications in computer graphics, scientific computing (finite element simulations) and surface modeling. If we view the quadrilateral mesh as a graph embedded on the surface (where each face is a quadrilateral), the ideal quadrilateral meshes are structured meshes where each interior vertex (i.e. a vertex not on the boundary on the mesh) belongs to 4 quadrilaterals and each boundary vertex belongs to 3 quadrilaterals. However, sometimes it is not possible to do so and one may be required to introduce *extraordinary* vertices that violate this condition. The natural problem then arises, how one can partition such an *unstructured* mesh into structured submeshes. This partitioning problem is of interest in the areas of mesh compression, mesh isomorphism and texture mapping.

In [1], Eppstein et. al. demonstrate how to partition quadrilateral meshes into structured submeshes using motorcycle graphs [3, 2]. If such a partition is represented using a *schematic partition*, they show that for bounded genus graphs the number of vertices, edges and faces of the schematic partition for the motorcycle graph is at most 24 times the optimal such schematic partition. They also show that computing an optimal schematic partition is NP-hard. They leave open several problems out of which we propose to investigate the following

- How hard is it to approximate the optimal structured partition? Is there a polynomial time approximation scheme to approximate such a partition to any degree of accuracy (and if so find one) or is there a corresponding hardness of approximation result for this problem?
- Can we prove tighter bounds on the approximation generated by the motorcycle graph based partitioning, since experiments show the results are quite good.
- The paper does not investigate graphs embedded on non-orientable manifolds. What can we say about quadrilateral meshes on such surfaces [The techniques in [1] only apply to graphs embedded on orientable manifolds].

References

- [1] D. Eppstein, M. T. Goodrich, E. Kim, and R. Tamstorf, "Motorcycle graphs: Canonical quad mesh partitioning" *Computer Graphics Forum*, vol. 27, 5(2008), 1477-1486.
- [2] S.-W. Cheng, A. Vigneron, "Motorcycle graphs and straight skeletons". *Algorithmica* 47, 2 (2007), 159-182.
- [3] D. Eppstein, J. Erickson, "Raising roofs, crashing cycles, and playing pool: applications of a data structure for finding pairwise interactions". *Discrete and Computational Geometry* 22, 4 (1999), 569-592.