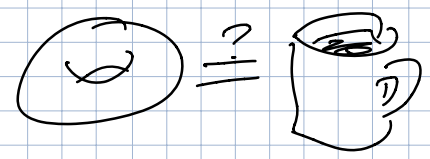
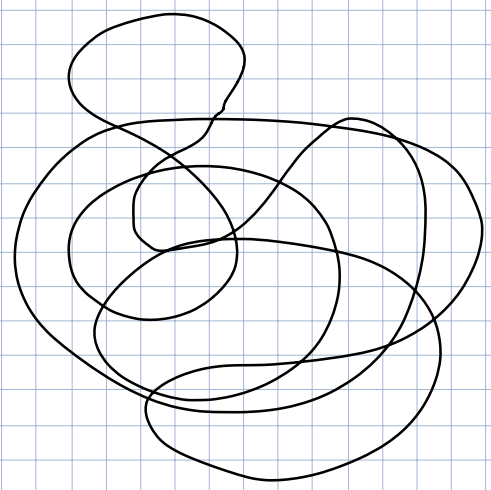
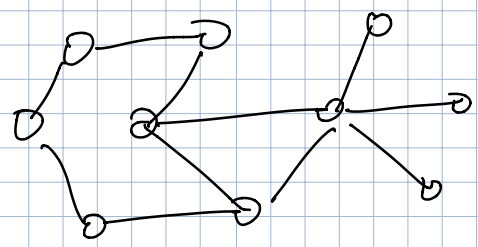
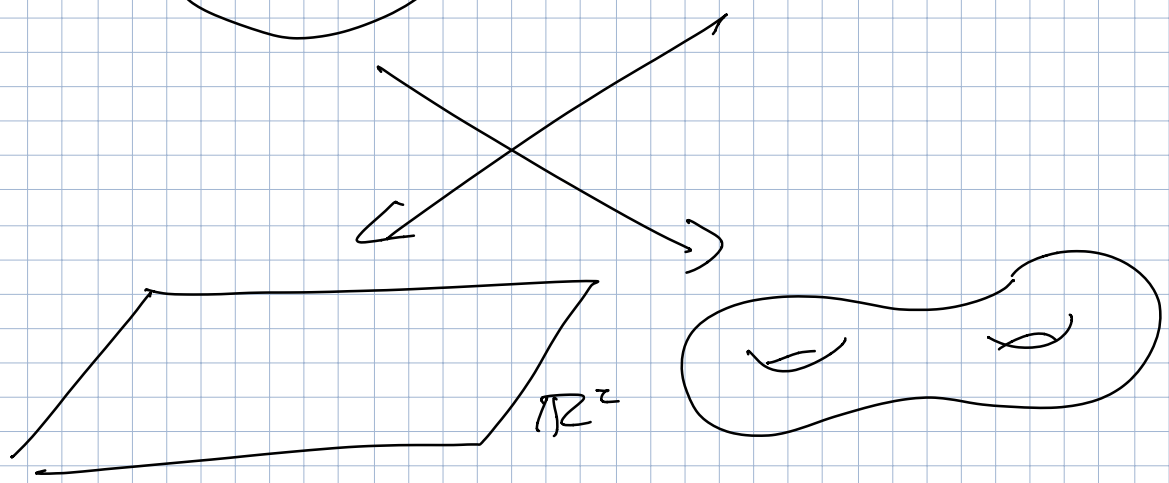
CS598 JGE

One-dimensional Computational Topology







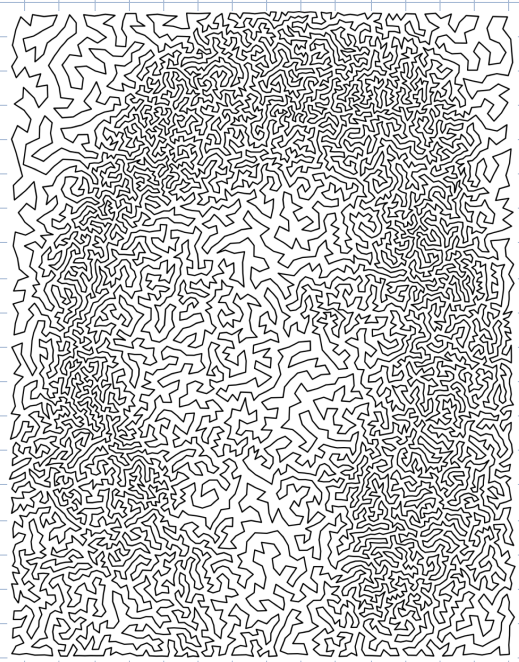


Homework

Project

- individual proposals
- group projects
  - report
  - presentation

Jordan Curve Theorem



Closed curve

$\gamma : S^1 \to \mathbb{R}^2$

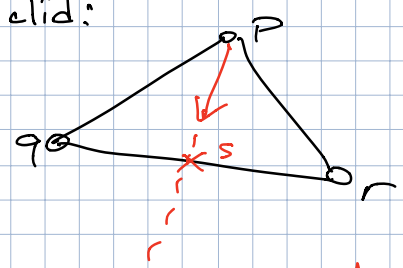
continuous

Simple : injective

JCT: For any simple closed curve γ in $\mathbb{R}^2$

$\mathbb{R}^2 \setminus \gamma$ has two components.

Euclid:



Bernard Bolzano – Antieuklid

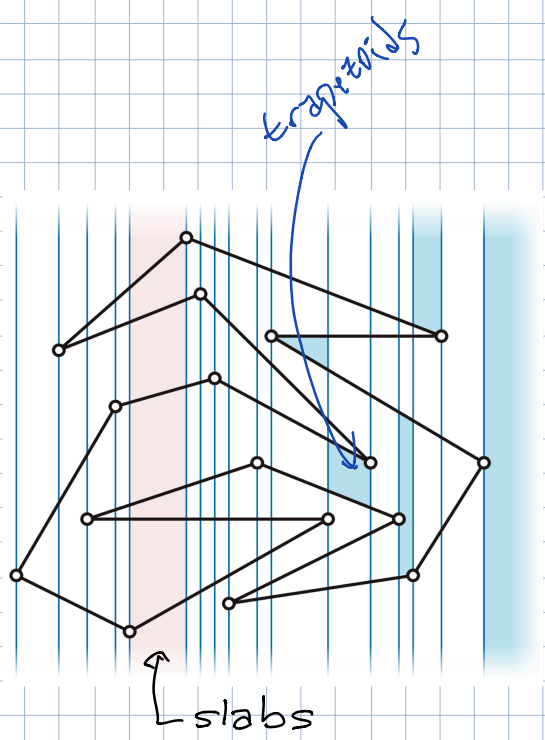
Emile Pasch

$\mathbb{R}^2 \setminus$ line is disconnected

Camille Jordan

Oswald Veblen ?

Thomas Hales 2005



Jordan Polygon Theorem

simple polygon P

PL simple closed curve

finite

$v_0, v_1, \ldots v_n = v_0$

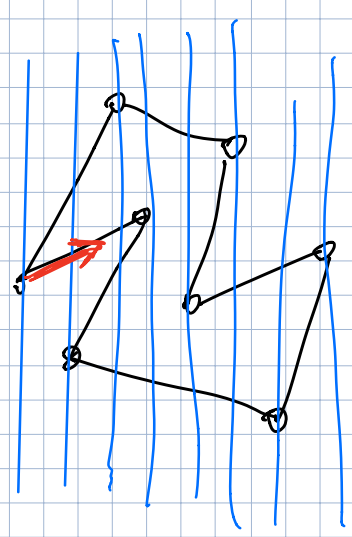
$\mathbb{R}^2 \setminus P$ has 2 components

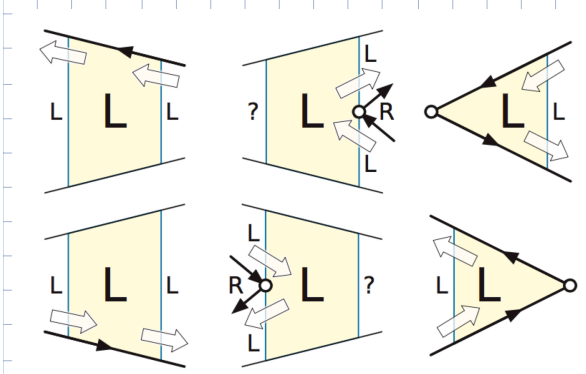
Schonflies 1901

Dehn 1899*

Lemma $\leq 2 \longrightarrow$ Left or Right

Lemma $\geq 2 \longrightarrow$ Odd or Even





Label L traps

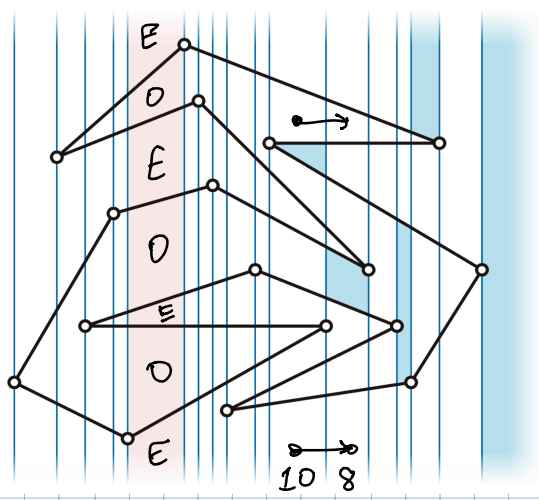
R traps

- Every trap is labeled
- All L traps connected

  R traps "

Topological invariant

Label each trap even or odd



Schoenflies: And the bounded component is a disk.

$h: \mathbb{R}^2 \to \mathbb{R}^2$

Gauss 1830

Eine interessante Aufgabe scheint zu sein, die Bedingung analytisch anzugeben, ob ein gegebener Punkt innerhalb oder ausserhalb der Figur fällt.

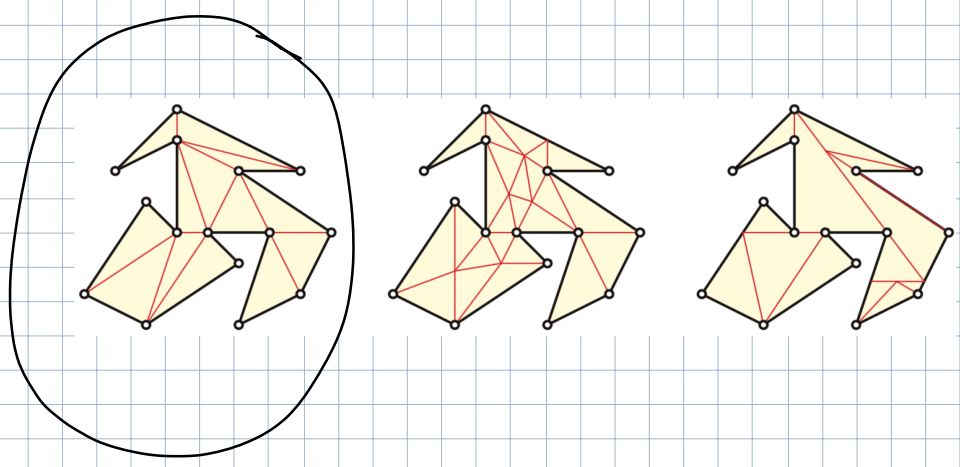
Die Auflösung ist leicht. Indem man den Punkt zum Anfangspunkt der Coordinaten wählt, zähle man alle Punkte

$$\begin{array}{llrrl} \alpha, & \text{wo} & y, & -y', & xy'-yx' \\ \beta, & \text{wo} & y, & -y', & yx'-xy' \\ \gamma, & \text{wo} & -y, & y', & xy'-yx' \\ \delta, & \text{wo} & -y, & y', & yx'-xy' \end{array}$$

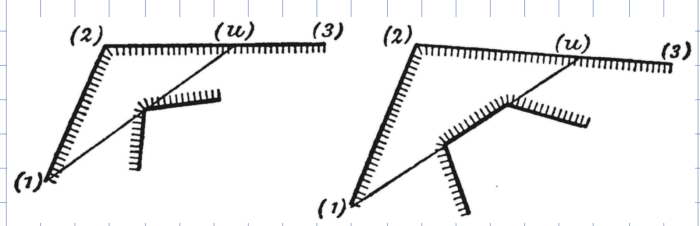
positiv sind; man hat dann

$$\alpha = \gamma, \qquad \beta = \delta.$$

Ist nun $\alpha - \beta = 0$, so liegt der Punkt ausserhalb, ist $\alpha - \beta = \pm 1$, so liegt er innerhalb.

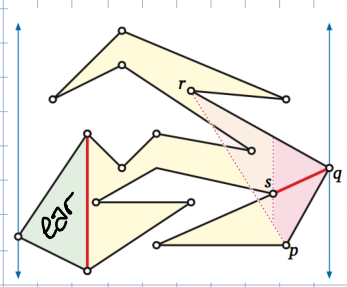


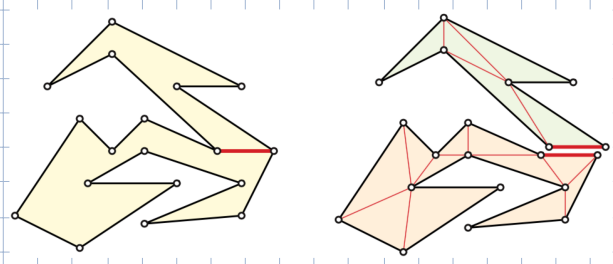
Gauss:

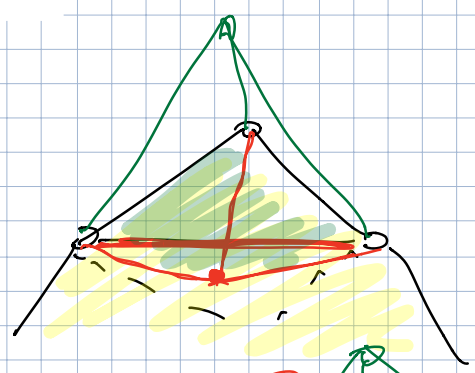


Diagonal

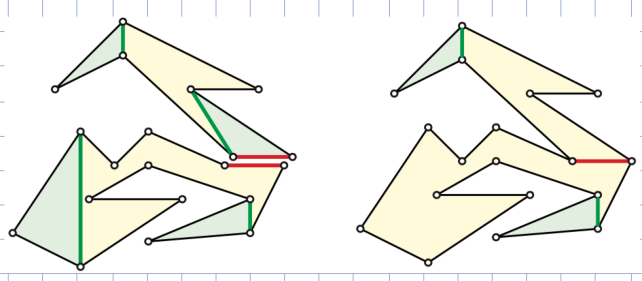
Dehn

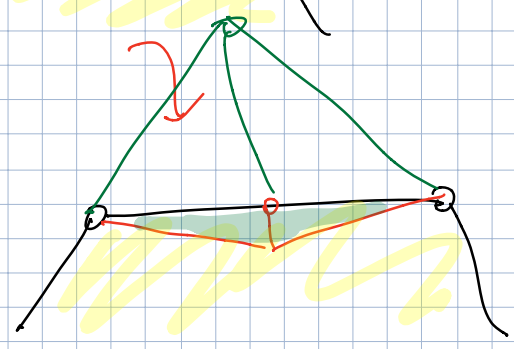


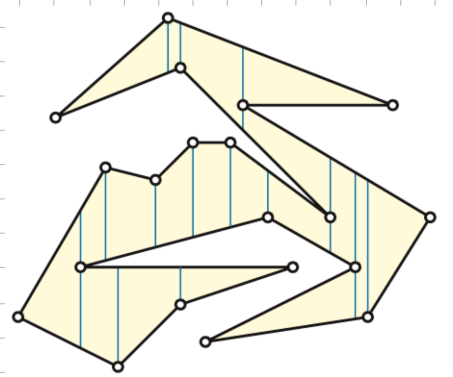




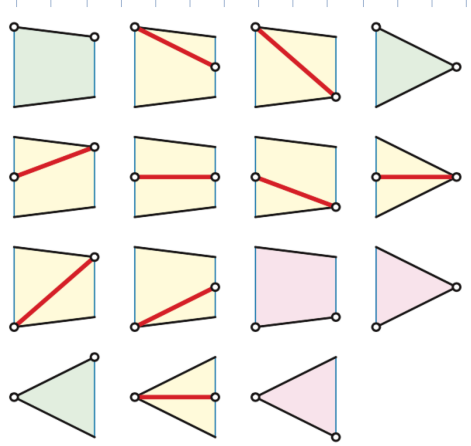
Two ears

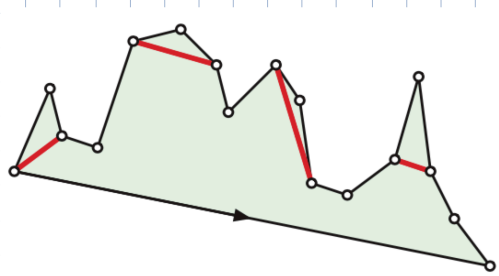






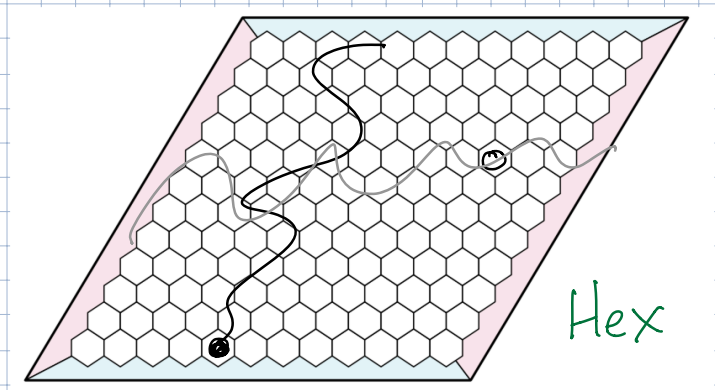
$O(n \log n)$





monotone mountains

$O(n)$



JCT $\Longleftrightarrow$ ~~At most~~ Exactly one player wins

one-page induction

Compactness

Brouwer Fixed point