

Admin: Travelling Sept 19 and 21 — no class
makeup lectures later

Office hours Fridays (when?) — Doodle
tomorrow 3-4 pm
3304 Siebel

Closed curves

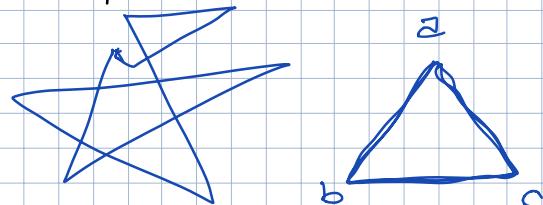
$\gamma: S^1 \rightarrow \mathbb{R}^2$ continuous

\mathcal{L} input to an algorithm

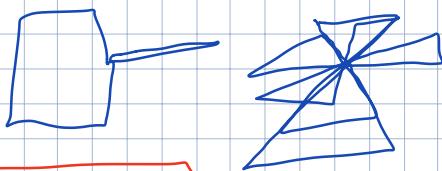
Three representations

- Polygons — specified by finite seq of pts p_0, \dots, p_{n-1}
 vertices
 $\text{edges } p_i p_{i+1 \text{ mod } n}$

Bradwardine 1340s

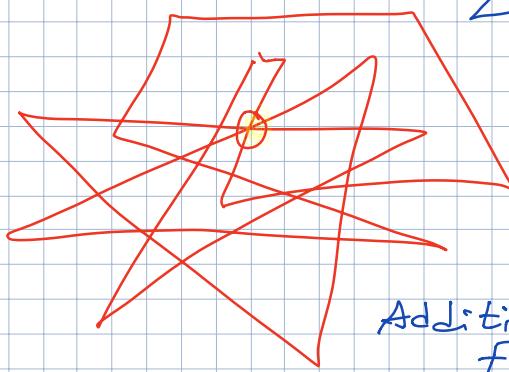
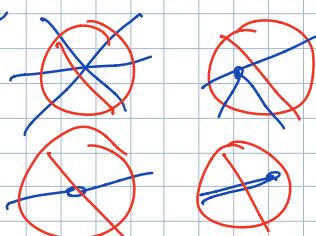
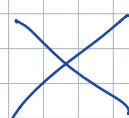


Meister 1700



abcabcabc

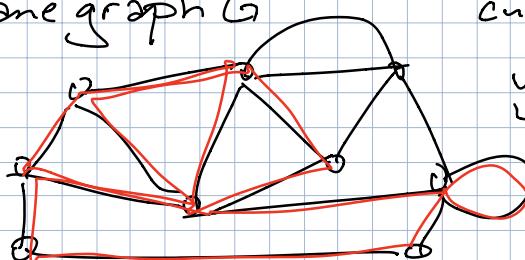
general position



Additional work to
find self-crossings

- Walk in plane graph G

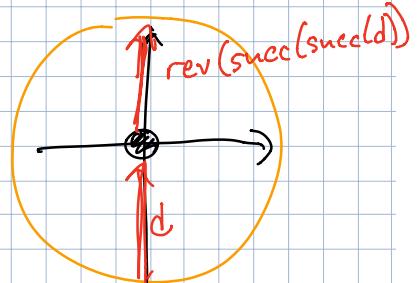
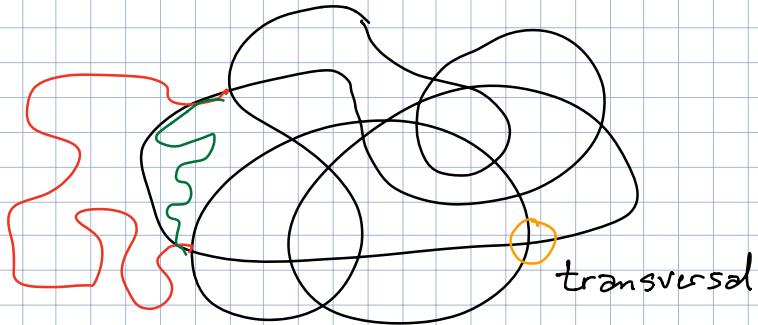
↑
geometrically
or
combinatorially



curve — closed walk

vevevevev

- Generic curves/immersions



- All self-intersections are
 - pairwise
 - transverse \rightarrow Finite #



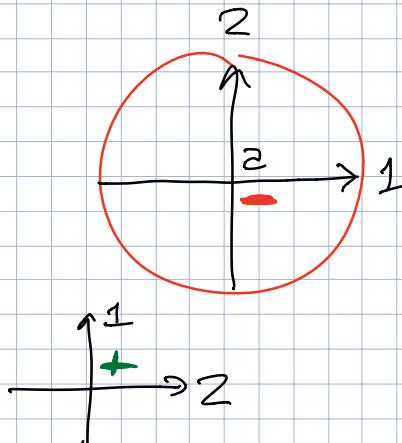
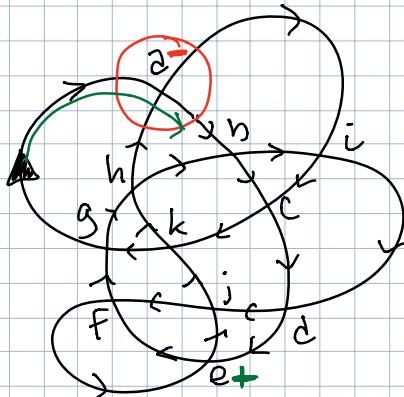
Every curve is arb. close to a generic curve

$$\gamma, \gamma': S^1 \rightarrow \mathbb{H}^2 \quad d(\gamma, \gamma') = \max_{\theta} \| \gamma(\theta) - \gamma'(\theta) \|$$

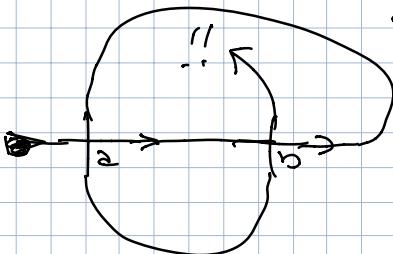
Represent by image graph 4-regular or cycle

or Gauss code (maybe signed)

abcedFghbidjfejkha⁺ckg



abab



$\bar{a} b c d e f g h b i d j f e \bar{j} k h \bar{k} i c \bar{k} g$

even

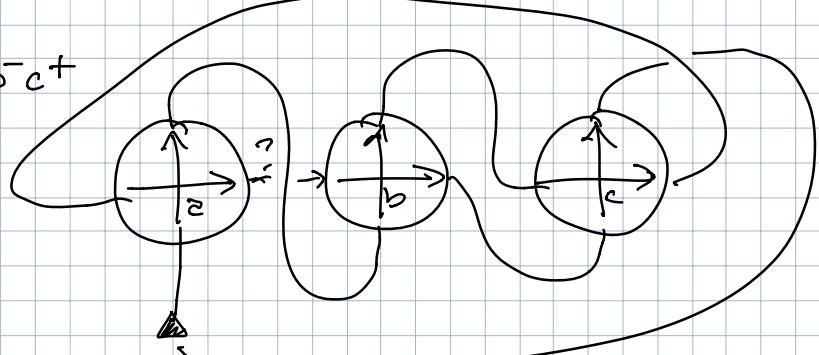
necessary condition

not suff:

$a b c a d c d e b e$

Gauss asked: When is a Gauss code planar?

$a^+ b^+ c^- a^- b^- c^+$

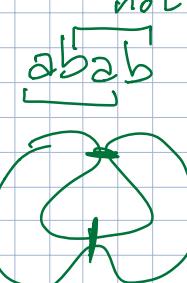


Signed Gauss code

embedding of graph Planar $\Leftrightarrow V - E + F = 2$
[Francis Carter 1970s]

Dehn's condition

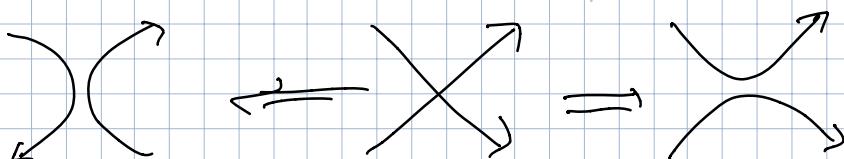
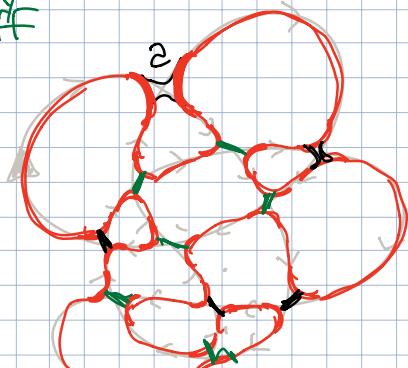
necessary
not suff

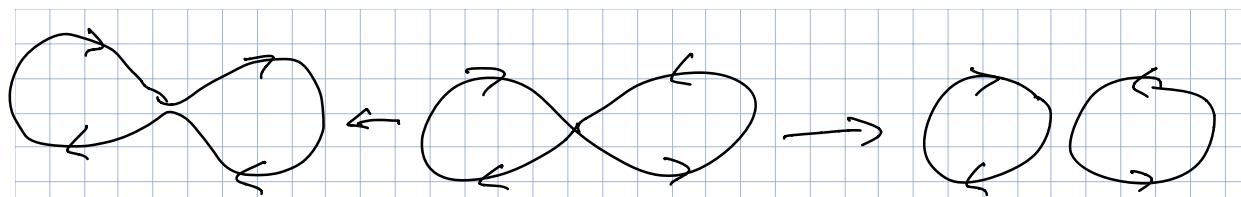


$\bar{a} b c \dots - f g k a q \dots - z$

rev

$\bar{a} k g f \dots - c b \bar{z} q \dots - z$





$a b c ? a d b c e d e$

Interleave graph: vertices = symbols

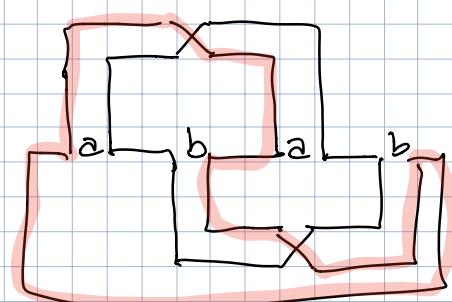
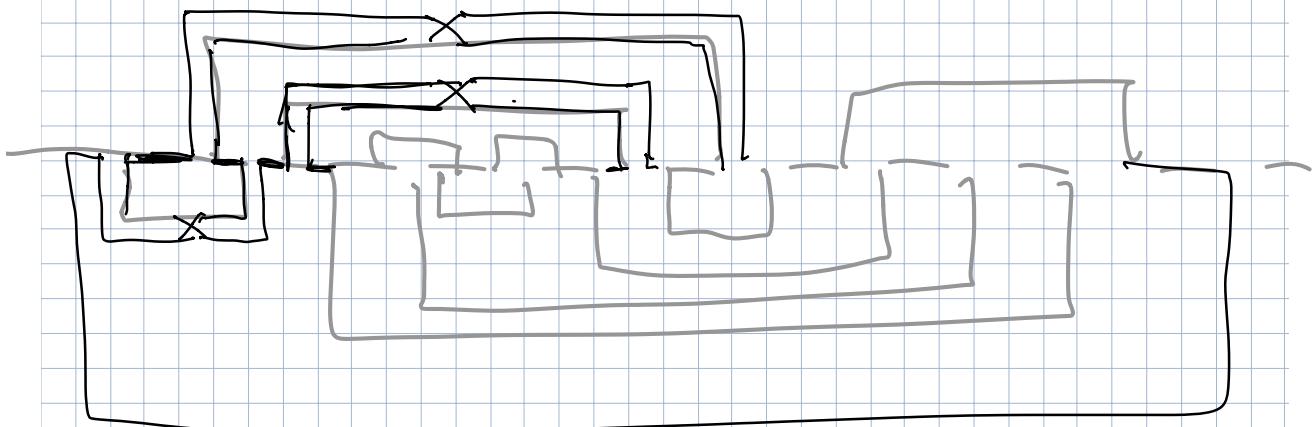
$$x - y \Leftrightarrow \begin{matrix} x & y & x & y \\ \swarrow & \searrow & \swarrow & \searrow \end{matrix}$$

planar \Leftrightarrow IG bipartite!

$O(n^2)$ time

Gauss + Dehn \Leftrightarrow planar Gauss code

Kaufman 1989



Interleave bipartite $\rightarrow O(n)$ time