

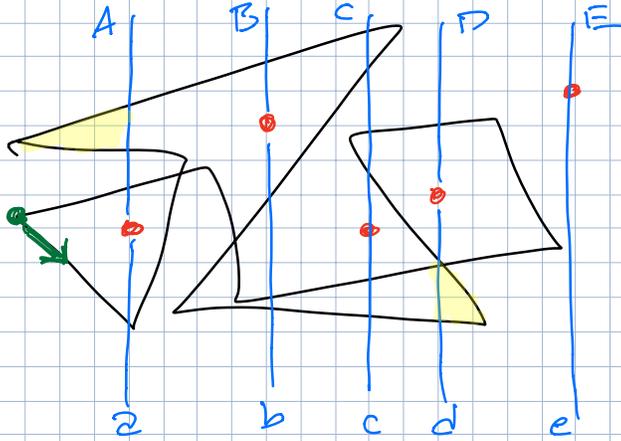
Last time:

points $p_1 \dots p_k$

~~Polygons P_1, P_2, \dots, P_k simple disjoint~~

polygon α

Is α contractible in $\mathbb{R}^2 \setminus P$?



~~a A B C C b b e d d C C D d~~

Reduce to $\varepsilon \Leftrightarrow$
contractible!

Sort P horizontally

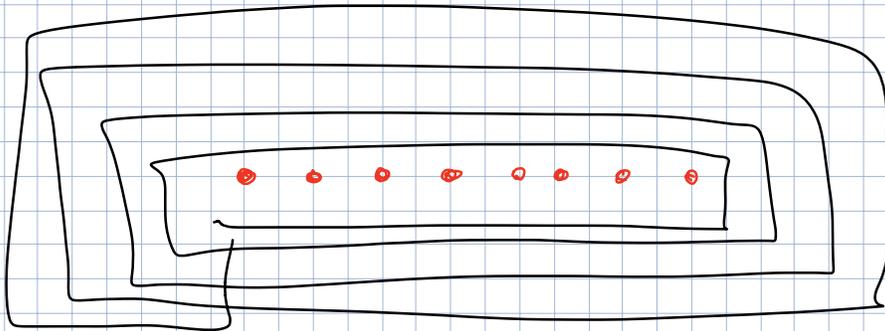
$O(k \log k)$

Compute crossing seq

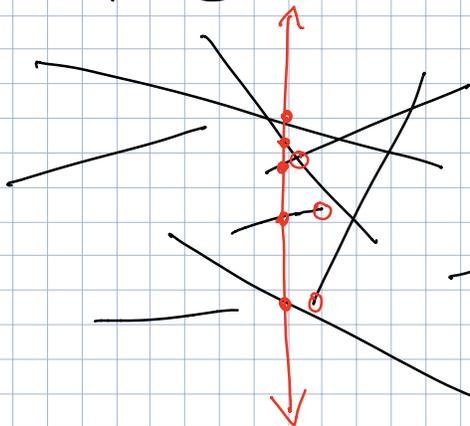
$O(n+x) = O(nk)$

Reduce crossing seq

$O(x) = O(nk)$



Plane sweep algorithm



Intuitively:
 Cont sweep vertical line L , maintain $L \cap S$ as a sorted seq of pts.

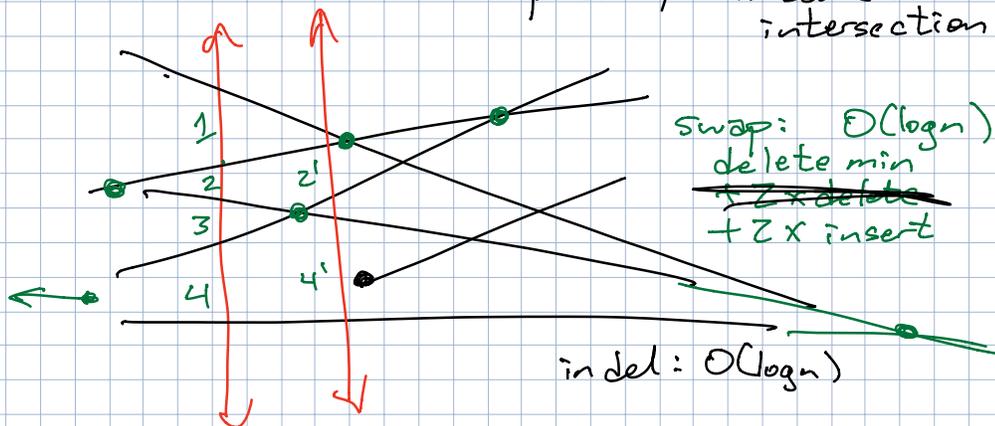
LNS changes:

- swap
- insert
- delete

Store LNS in a balanced BST — each update in $O(\log n)$ time

Process events in order \rightarrow

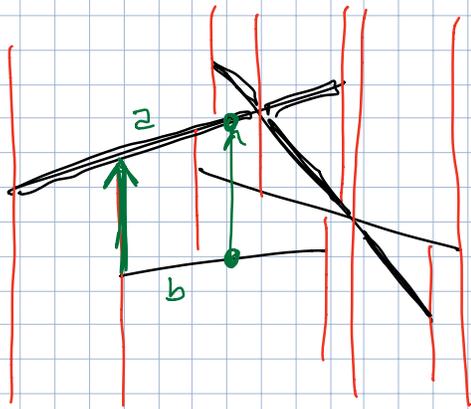
- Sort endpts by x-coords
- Keep possible next swaps in a p.queue
 priority = x-coord of intersection



$$O((n+x) \log n) = O(n^2 \log n)$$

\uparrow #segs \uparrow #ints

More work: $O(n \log n + x) = O(n^2)$



Elementary segments
between endpoints
+ crossing pts

$a \uparrow b$ "a is above b"

Claim: \uparrow is acyclic

Proof:

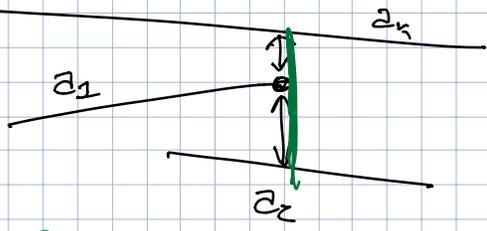
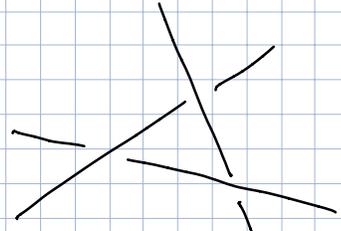
Smallest cycle

~~$a_1 \uparrow a_2 \uparrow \dots \uparrow a_k \uparrow a_1$~~

$k=1$ impossible

$k=2$ impossible

$k \geq 3$ a_1 has leftmost
right end



$a \uparrow b$ some vertical fence
connects a to b
and $a \uparrow b$

$\uparrow^* = \uparrow^*$ acyclic

we can extract a dag
whose verts are el. segs
and whose edges \uparrow^*

Global vertical ranking of el segs and obstacles



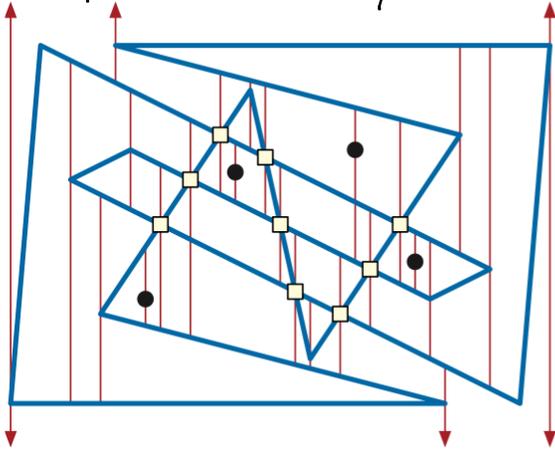
collapse segs between obstacles

- \rightarrow rank(seg) odd
- \rightarrow rank(obst) even

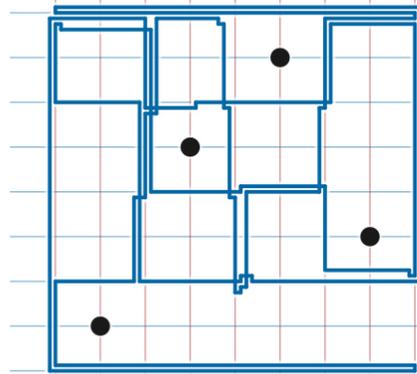
all ranks $1 \dots 2k+1$

Similar horiz ranking by x-coords

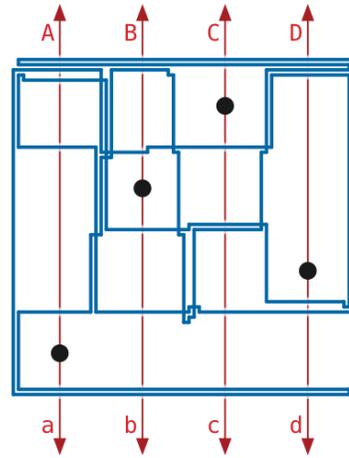
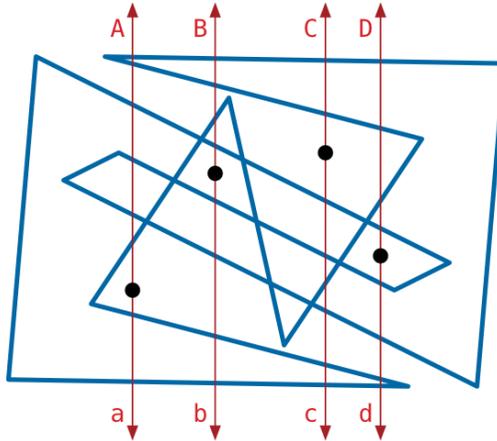
trapezoidal decomposition



replace coords by ranks



"rectified"



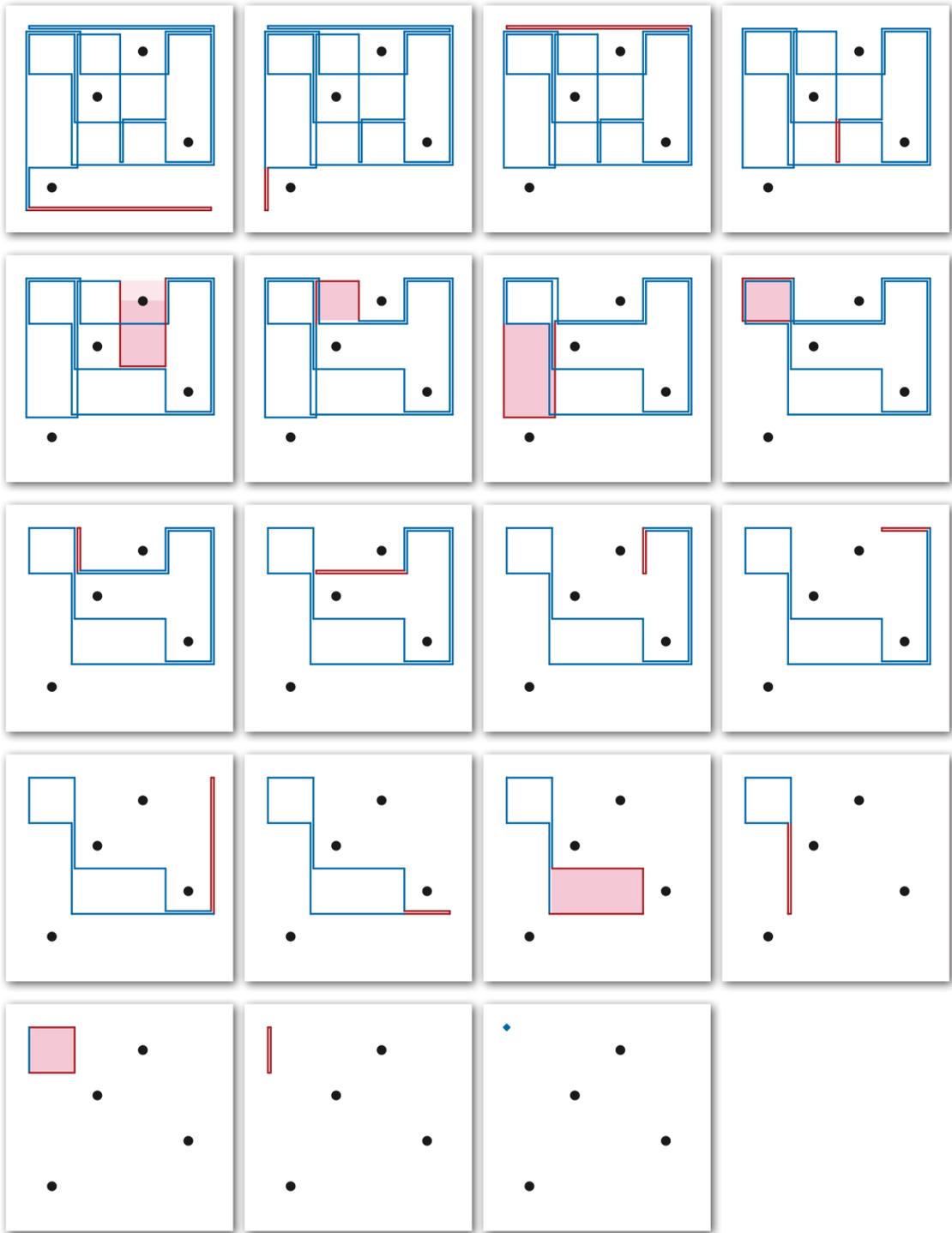
same crossing sequence

Either both contractible or neither

Reduce by sliding brackets

- Changes x seq by elem. reductions

We can always make progress unless
the polygon isn't contractible



For every edge

Is it a bracket? $\square \cup \supset \cup$ not $\lrcorner \llcorner$

Is it slidable? *

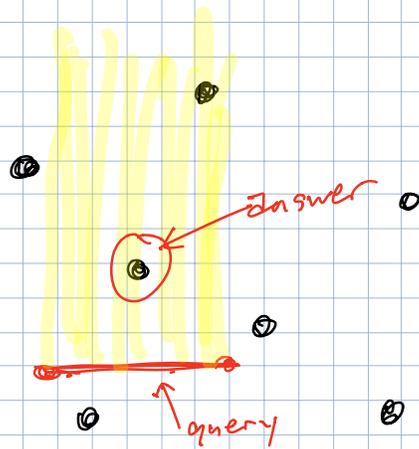
If not mark it FROZEN

Keep a bag of unfrozen edges

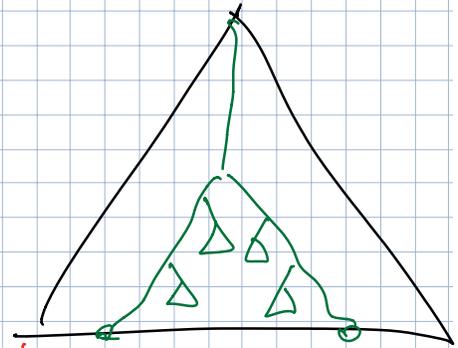
Always unfrozen edge available until done

bag ins/del = $O(n)$

Remains: Bracket slides



layered range tree
BST over
x-coords



$O(\log^* n)$ query time

$$\Rightarrow O((n+k+s) \log(n+k+s))$$