

# Projects ~

Solo proposals 2-3 pages

3 weeks

What do you want to do?

Background — Refs / sketch of known results

Proposal — half-baked is okay

Group projects up to 3

— presentation ~20 min

— writeup ~15 pages

} Finals week

## Planar graph algos — MSSTP

negative-lengths SP

Today: minimum cuts  
maximum flows

Separators

r-divisions

Monge structure

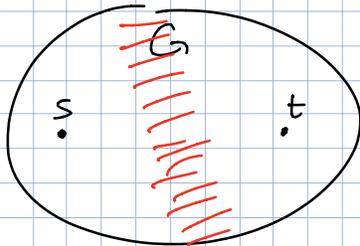
Monge heap

Crossing arguments

winding arguments

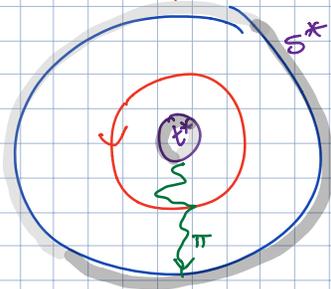
## Min cuts

Edges of  $G$  have  $\leftarrow$  non-negative weight  $\rightarrow$   
(capacity)



We want min-weight (s,t)-cut

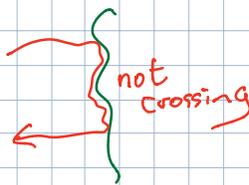
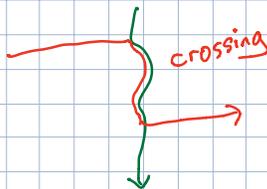
Duality — simple cut  $\Leftrightarrow$  simple cycle  
separates  $s$  and  $t$  separates  $s^*$  and  $t^*$



Shortest cycle in an annulus  
separates boundary  
winding #1 around  $t^*$   
homotopic to boundary

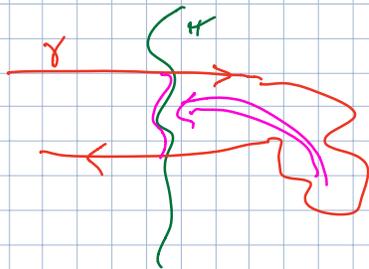
$\pi$  = shortest path from  $s^*$  to  $t^*$

Intuition: min cycle crosses  $\pi$  once

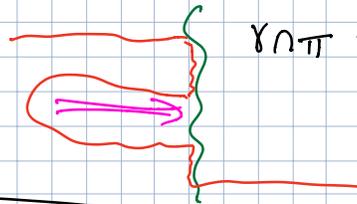


Define  $x$ -seq: write  $+$  whenever  $\gamma$  crosses  $\pi \rightarrow$   
 $-$   $\leftarrow$

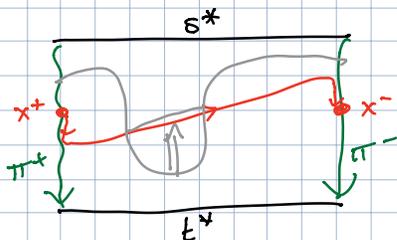
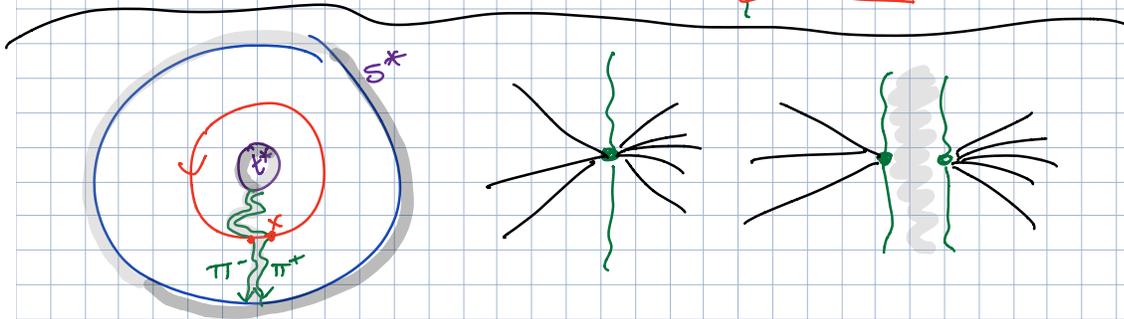
Reduced  $x$ -seq =  $+$  Either reduced or contains  $+-$   
 or  $-+$



IF  $\gamma$  not reduced then not shortest



$\gamma \cap \pi$  is a subpath of  $\pi$



shortest path from  $x^+$  to  $x^-$   
 for some vertex  $x \in \pi$   
 best

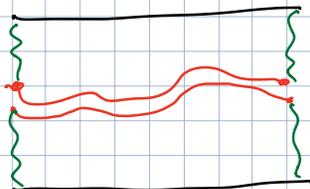
$k = \#$  vertices in  $\pi$

Naive: Dijkstra  $\times k \Rightarrow O(k n \log n) = O(n^2 \log n)$

MSSP  $\Rightarrow O(n \log n)$

Divide & conquer (Reif)  $\Rightarrow O(n \log n \log k)$

Compute shortest path from  $x_{k/2}^+$  to  $x_{k/2}^-$   
 cut rect along that path  
 recurse in both parts



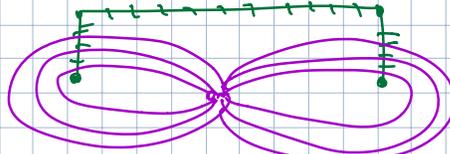
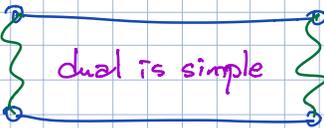
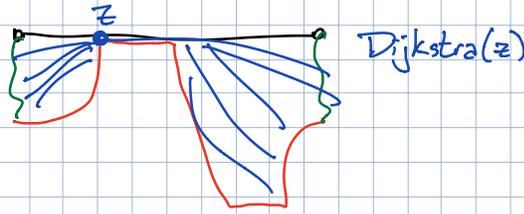
$n = \#$  faces Euler  $\Rightarrow \#$  vertices  $= \Theta(n)^*$

$$T(n, k) = O(n \log n) + T(n_1, \frac{k}{2}) + T(n_2, \frac{k}{2})$$

$n_1 + n_2 = n$

$$= O(n \log n \log k)$$

Caveat:



Italiano et al. [2011]  $\leadsto O(n \log \log n)$

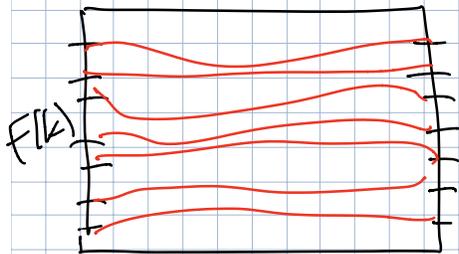
$$O(n \log \log k)$$

$O(n \log \log n)$  prep.

compute  $\log^2 k$  LR shortest paths in  $O(n)$  time

recurse\* in each stripe  
\* with early stopping

$$T(k) = O(n) + \sum_{i=1}^{F(k)} T(n_i, \frac{k}{F(k)})$$



$$\text{Let } F(k) = \frac{n}{F(k)}$$

$$T(n, k) = O(n F^*(k))$$

FR - Bellman Ford

Exploits Monge structure

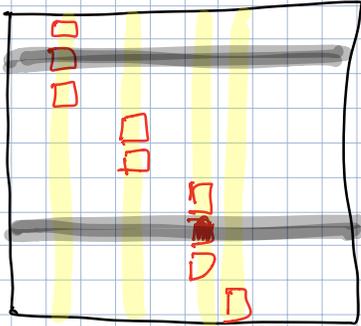
FR - Dijkstra

DDG  $O(\frac{n}{r})$  vertices and  $O(n)$  edges

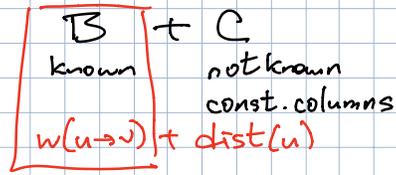
$$\text{Dijkstra: } O(E + V \log V) = O(n + \frac{n}{r} \log n)$$

BAD

# "Monge heap"



M - unknown Monge array



- Reveal a column
- Find min revealed entry
- Kill a row