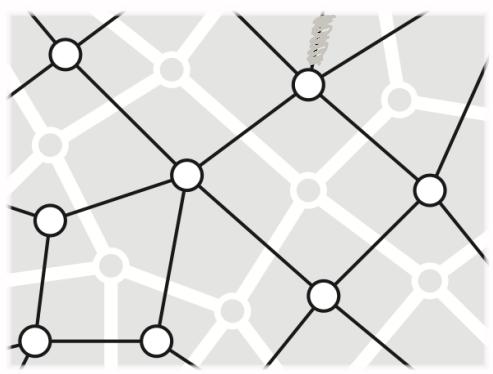
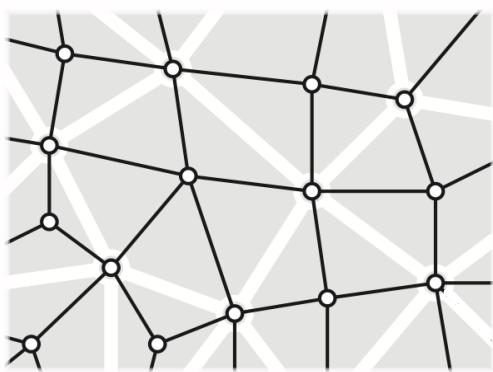


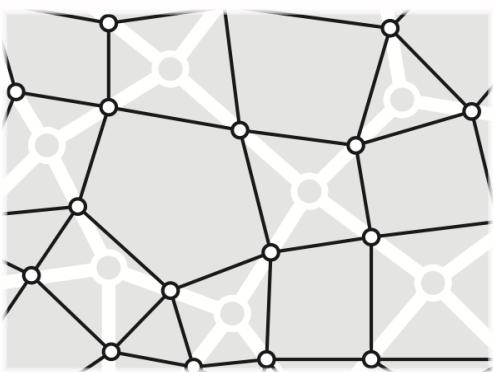
A region of a surface map  $\Sigma$



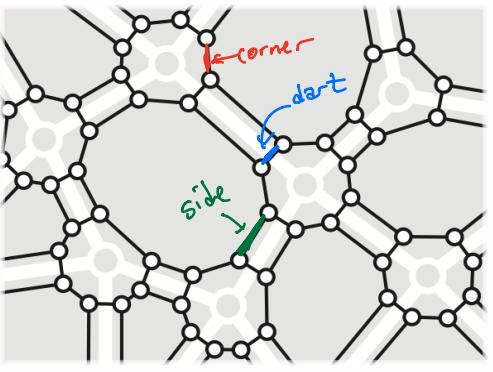
The corresponding region of the dual map  $\Sigma^*$



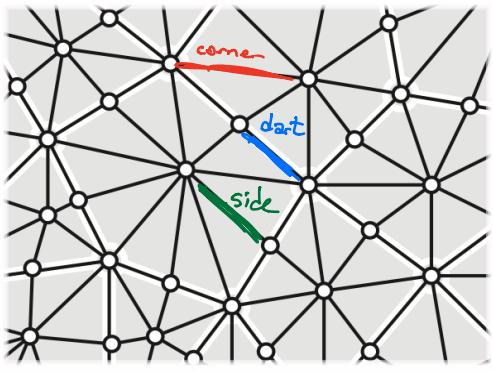
The radial map  $\Sigma^\diamond$



The medial map  $\Sigma^x$



The band decomposition  $\Sigma^\square$



The barycentric subdivision  $\Sigma^+$

primal $\Sigma$	dual $\Sigma^*$	primal $\Sigma$	dual $\Sigma^*$
vertex $v$	face $v^*$	underlying surface $ \Sigma $	underlying surface $ \Sigma $
edge $e$	edge $e^*$	polygonal schema	rotation system
face $f$	vertex $f^*$	reflection system $(\Phi, a, b, c)$	reflection system $(\Phi, c, b, a)$
blade $\phi$	blade $\phi^*$	band decomposition $\Sigma^\square$	band decomposition $\Sigma^\square$
dart $d$	side $d^*$	medial map $\Sigma^x$	radial map $\Sigma^\diamond$
corner $c$	corner $c^*$	boundary loop	bridge
side $s$	dart $s^*$	loop	isthmus
$\text{tail}(d)$	$\text{left}(d^*)$	cycle	cocycle
$\text{head}(d)$	$\text{right}(d^*)$	boundary subgraph	edge cut
$\text{left}(s)$	$\text{tail}(s^*)$	deletion $\Sigma \setminus e$	contraction $\Sigma^* / e^*$
$\text{right}(s)$	$\text{head}(s^*)$	contraction $\Sigma / e$	deletion $\Sigma^* \setminus e^*$

orientable only!

Correspondences between features of primal and dual surface maps

## Reflection system $(\underline{\Phi}, \underline{a}, b, c)$

$\underline{\Phi}$  — set of blades / flags

dual:  $(\underline{\Phi}, c, b, a)$

$a, b, c$  — involutions of  $\underline{\Phi}$

such that  $a \circ c = c \circ a$

vertices = orbits of  $\langle b, c \rangle$

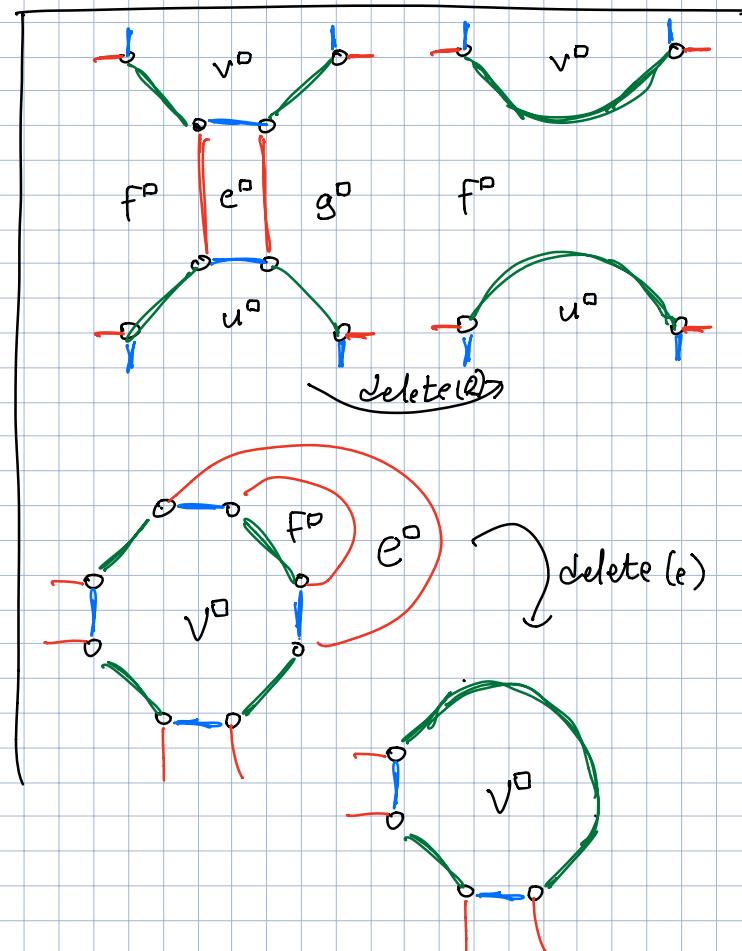
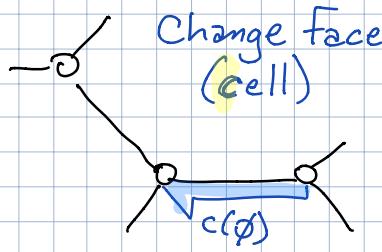
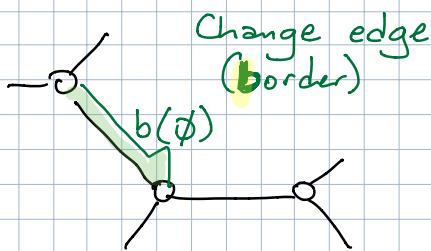
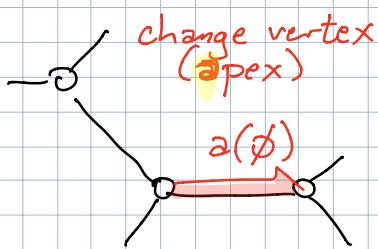
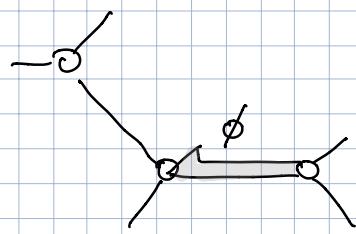
edges = orbits of  $\langle a, c \rangle$

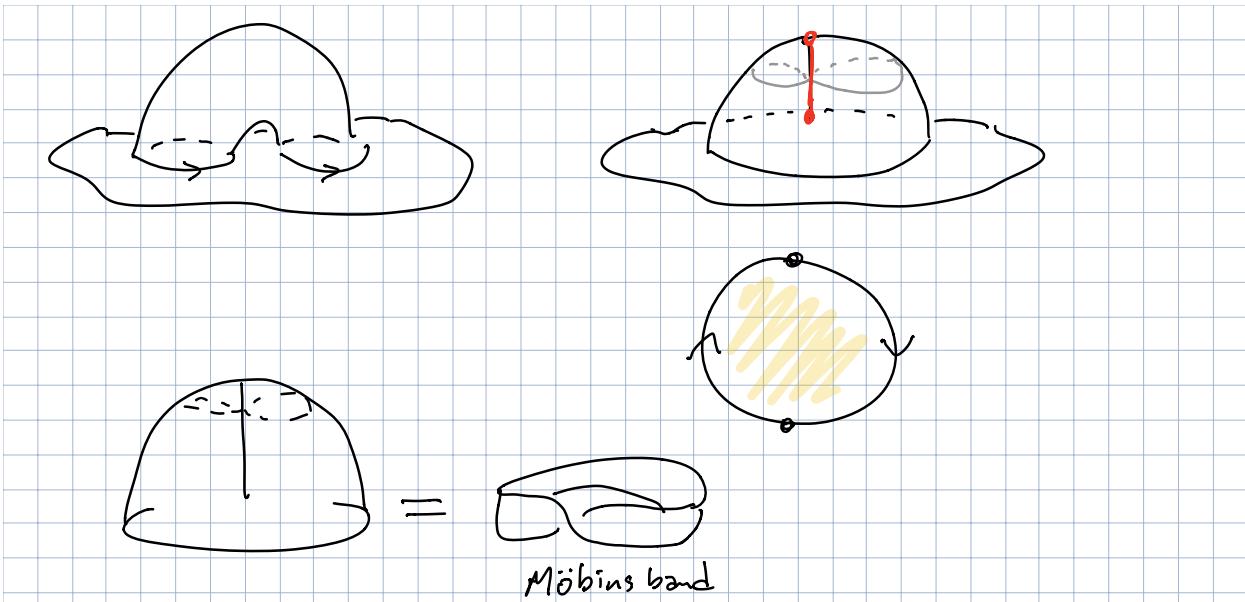
faces = orbits of  $\langle a, b \rangle$

sides = orbits of  $a$

corners = orbits of  $b$

darts = orbits of  $c$



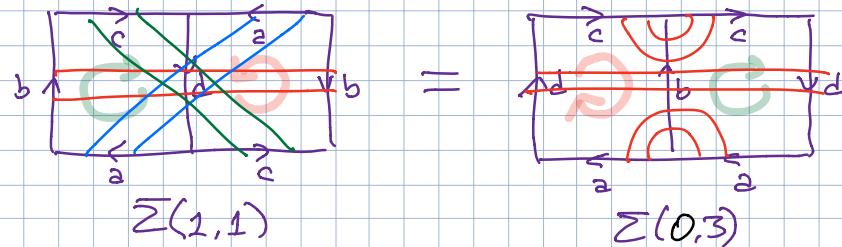


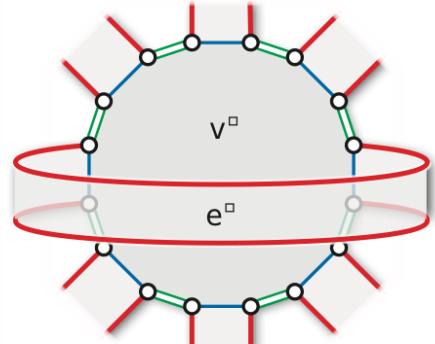
### Classification Theorem:

Every surface "is" a sphere with  
 $g \geq 0$  handles ~~and or~~  $h \geq 0$  twists attached  $\Sigma(g,h)$

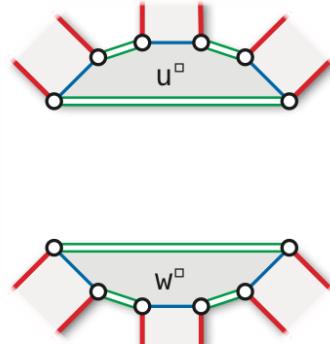
$$\text{Euler: } V - E + F = 2 - 2g - h$$

$$\text{Dyck: } \Sigma(g,h) = \Sigma(g-1, h+2) \text{ if } h \geq 1$$

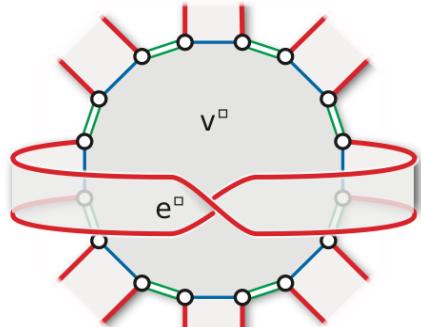




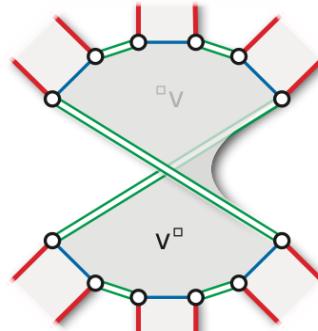
*contract*  
*expand*



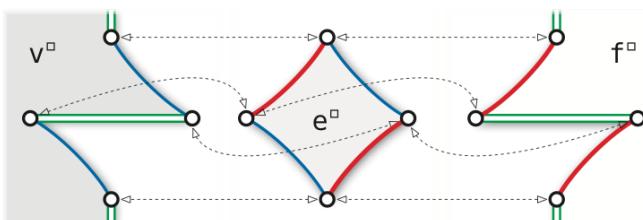
Detaching a handle by contracting an two-sided non-separating loop.



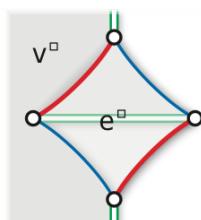
*contract*  
*expand*



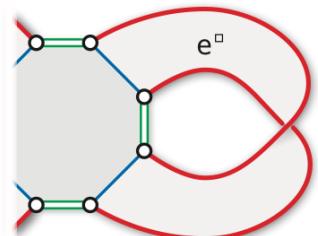
Detaching a twist by contracting a one-sided loop.



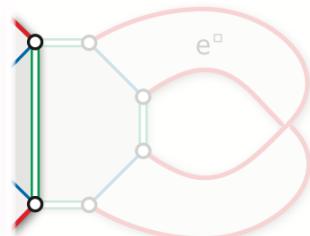
=



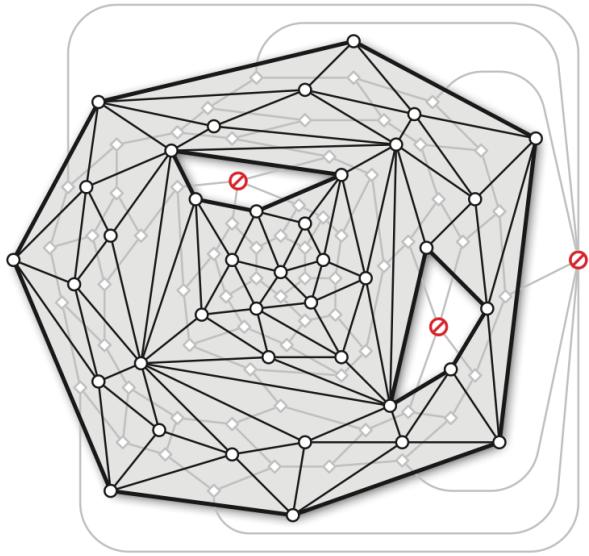
A self-dual twist.



*delete*  
*insert*  
*contract*  
*expand*



Contracting or deleting a self-dual twist.



## Surfaces with bdry

Combinatorially:  
 comb. map  
 with faces marked "gone"  
 ↗ disjoint!

G: holes  
 $G^*$ : punctures  
 ↗ closed disk  
 ↗ open disk



## Tree-cotree decomposition

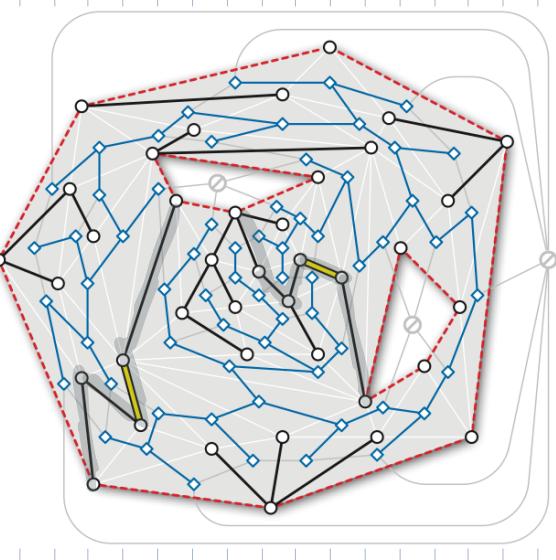
$$(T, F, L)$$

T = spanning tree of G

F = spanning forest of  $G^*$   
 with one tree per puncture

$$L = E \setminus (T \cup F)$$

$$|L| = 2g + h + b - 1$$



## Forest-cotree decomposition

$$(\partial G, F, C, L)$$

$\partial G$  = boundary edges

C = spanning tree of  $G^* \setminus$  punctures

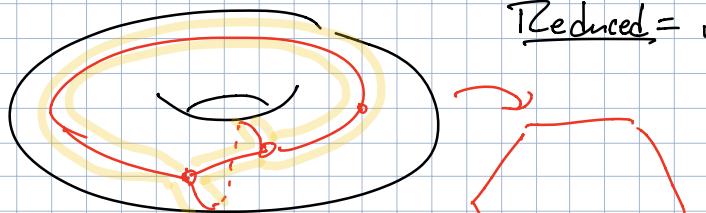
F = spanning forest of G  
 with each tree containing  
 one vertex on boundary

$$L = E \setminus (\partial G \cup C \cup F)$$

$$|L| = 2g + h + b - 1$$

Cut graph = subgraph of  $G$

that cuts surface into a disk



Reduced = no degree-1 verts.

$(T, L, C)$  - any tree-cotree  
decomp.

$T \cup L$  = cut graph

↓ reduce by removing "hair"

