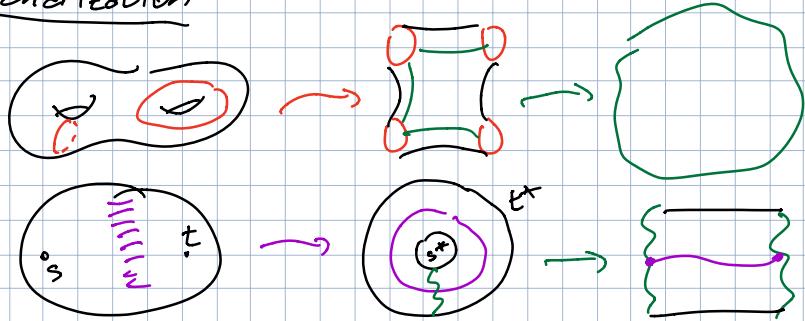
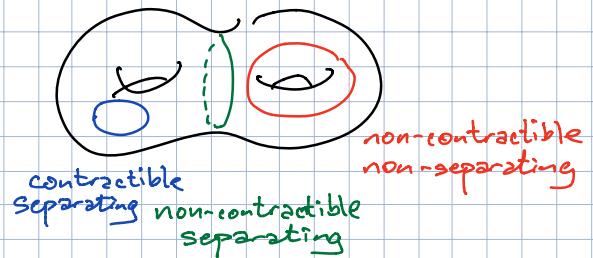


Planarization



Find shortest interesting cycles in surface graph with generic non-~~reg~~ weighted edges



contractible — trivial homotopy
separating — trivial homology

Also: "topological noise"

Thomassen's 3-path condition [90]

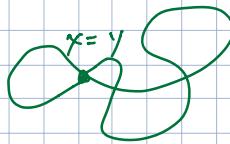
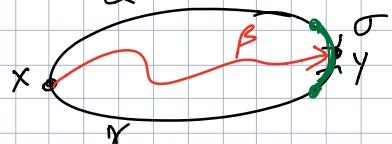


If $\alpha \cdot \text{rev}(\beta)$ is trivial and $\beta \cdot \text{rev}(\gamma)$ is trivial then $\alpha \cdot \text{rev}(\gamma)$ is trivial

$\Rightarrow \sigma = \text{shortest nontrivial cycle}$

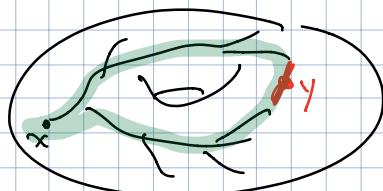
$x, y = \text{antipodal points on } \sigma$

α, γ are shortest paths.



$\sigma = \text{cycle}(T, e)$

where $T = \text{shortest path tree}@x$
 $e \notin T$



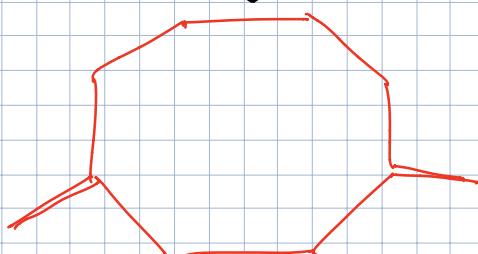
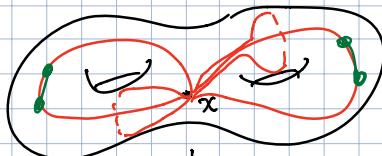
for all x
for all $e \notin T_x$
if $\text{loop}(T, e)$ is nontrivial

$O(n^3)$ time

Greedy tree-cotree decomposition (T, L, C)

- T = shortest path tree rooted @ x $\leftarrow O(n \log n)$
- C = min. spanning tree of G^* where $w(e^*) = l(\text{loop}(T, e))$ $\leftarrow O(n \log n)$
 $\leftarrow O(n)$
- $L = E \setminus (C \cup T)$

$$L = \{\text{loop}(T, e) \mid e \in L\} \quad \text{— greedy system of loops}$$



• every loop in L is non-sep

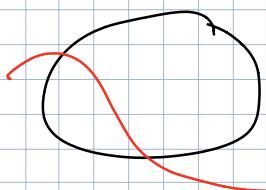
• L is a basis for $\pi_1(\Sigma)$

any closed walk thru x is homotopic to a concat of loops in L

• L is the shortest system of loops with basepoint x . [EWDS, CID]

• every loop in L is composed of two sh. paths and an edge

Shortest nontrivial cycle crosses any shortest path at most once

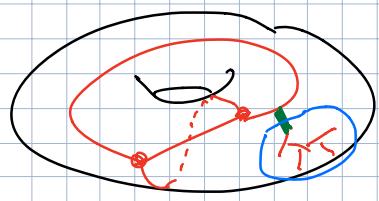


Shortest nontrivial cycle crosses some loop in L at least once

$\Rightarrow g^{O(n)}$ in time. [Kutz]

Dual cut graph:

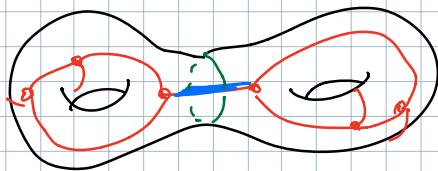
$$K^* = C^* \cup L^* \subseteq G^* \quad \text{cuts } \Sigma \text{ into a disk}$$



Reduced R^* — repeatedly remove deg-1 vertices from K^*

↑
"hair"

Lemma: $\text{cycle}(T, e)$ is separating $\Leftrightarrow e^*$ is a bridge in K^*
 $\text{cycle}(T, e)$ is contractible $\Leftrightarrow e^*$ is hair



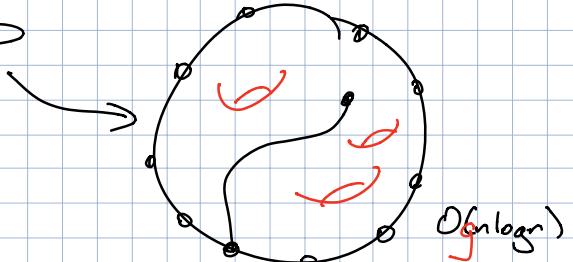
Shortest non-con cycle is $\text{cycle}(T, e)$
 for some $e^* \in T^*$

non-sep cycle is $\text{cycle}(T, e)$

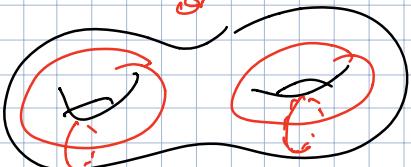
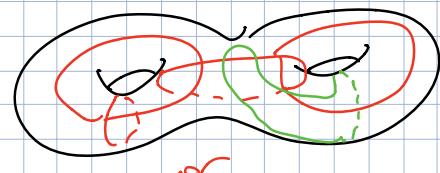
for some e^* in 2-conn comp
 of T^*

\Rightarrow shortest non-trivial cycle thru x in $O(n)$ time
 shortest overall in $O(n^2)$ time [EH(03, ++)]

$O(g^2 n \log n)$ time via MSSP

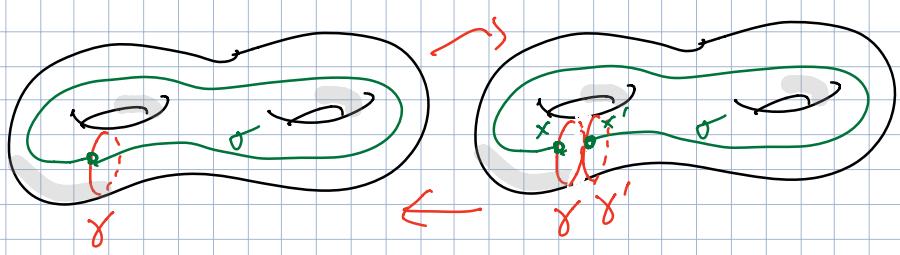


(T, L, C) — greedy
 $\Gamma = \{\text{cycles}(T, e) \mid e \in L\}$



- Shortest
- Every non-sep cycle crosses every cycle in Γ at most once.
 - Every non-sep cycle crosses some cycle in Γ odd # times
 - Every cycle crosses something in Γ odd # times is non-sep

Shortest non-sep cycle is
 the shortest cycle crossing
 some $\gamma \in \Gamma$ exactly once



$\min_{x \in \gamma} \text{dist}(x, x')$ in $G \otimes \gamma$
 $O(gn \log n)$ time MSSP
