

Friday 12-15 - watch web site for location

MSSP \rightarrow Min Cut \leftarrow Homology

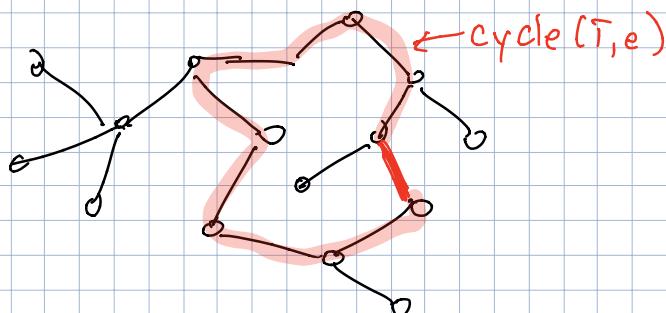
Cycle space — elements are even subgraphs

$$= \bigoplus \text{cycles}$$

vector space wrt xor

$$\mathbb{Z}_2^{E-V+1}$$

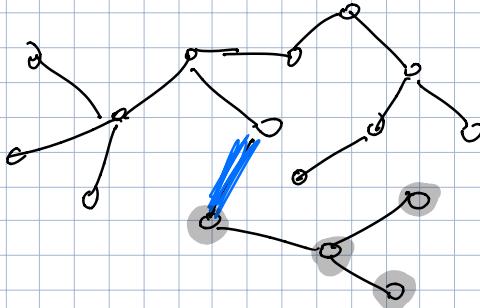
Basis = fundamental cycles wrt any spanning tree



Cut space — elements are edge cuts

$$\text{vector space } \mathbb{Z}_2^{V-1}$$

Basis = fundamental cuts wrt spanning tree



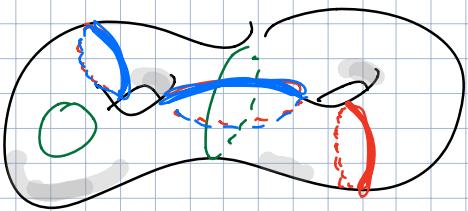
Cuts and cycles are orthogonal.

Planar graphs: $\text{Cycle}(G) = \text{Cut}(G^*)$

$$\mathbb{Z}^{E-V+1} = \mathbb{Z}^{F-1}$$

Surface graphs?

cuts \Leftrightarrow up to $g+1$ cycles



Boundary subgraph = boundary of a subset of faces

Every bdry subgraph is even

Boundary space = \mathbb{Z}^{F-1}

Two subgraphs A, B are homologous

iff $A \oplus B$ is a boundary

Homology space = $\mathbb{Z}/B = \mathbb{Z}_2^{\text{sg}}$

= cycle space / bdry space

= $H_1(\Sigma, \mathbb{Z}_2)$

First homology group of surface Σ
with \mathbb{Z}_2 coefficients.

Tree-cotree decomp. (T, L, C)

Let $\Pi = \{\text{cycle}(T, e) \mid e \in L\}$ $\gamma_1 \dots \gamma_{2g}$

These cycles form a basis for H_1

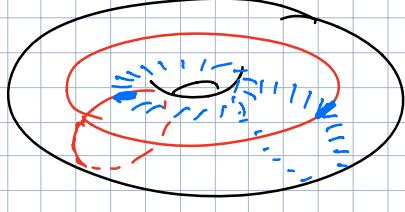
Every even subgraph is homologous with
 \oplus cycles in Π

Cohomology — Homology in G^* — Computing homology

Let $\Lambda = \{\text{cocycle}(G, e) \mid e \in L\}$

$\lambda_1, \lambda_2 \dots \lambda_{2g}$

$L = \{e_1 \dots e_{2g}\}$



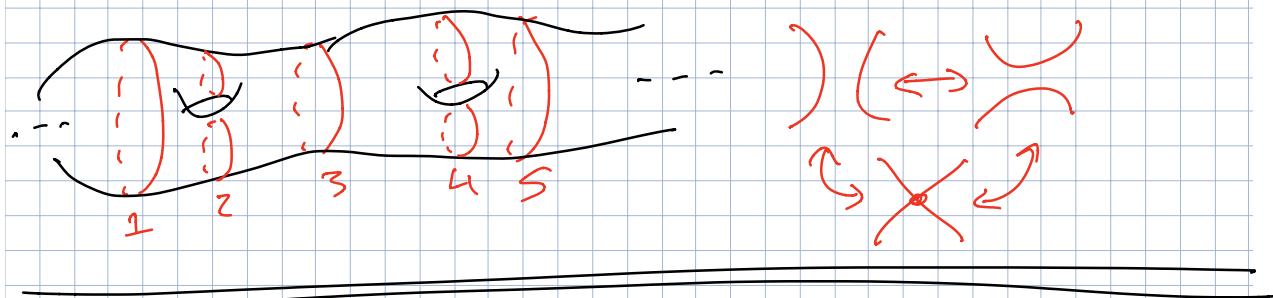
Signature $[e]$ = vector of $2g$ bits

i^{th} bit = $[e \in \lambda_i]$

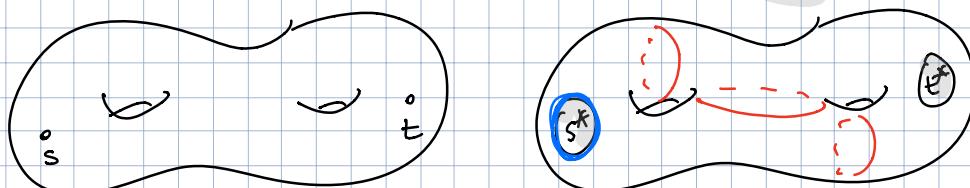
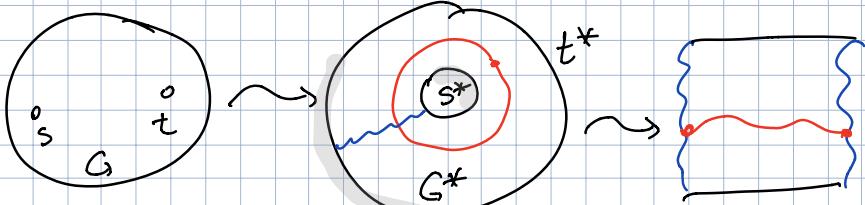
Lemma: Two subgraphs $A + B$ are homologous

$$\Leftrightarrow [A] = [B] \quad \text{where } [A] = \sum_{e \in A} [e]$$

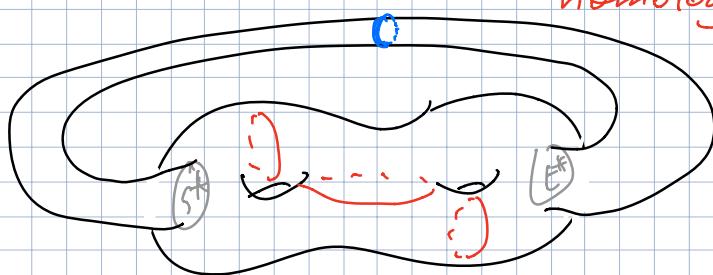
$$\Leftrightarrow [A \oplus B] = 0$$



Minimum Cuts



minimum (s, t) -cutting = min wt even subgraph in G^* homologous with ∂s^*



Reduced to following: Given a cycle γ on surface graph with weighted edges

Find min wt even subgraph γ' s.t.
 $[\gamma] = [\gamma']$

Bad news: This is NP-hard

Good news: FPT ($F(g) \cdot n \log \log n$)

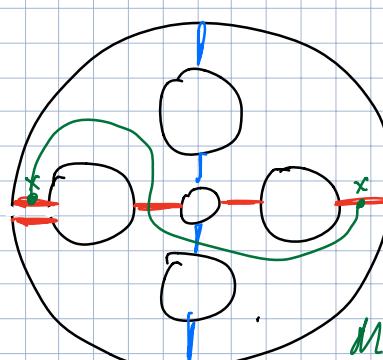
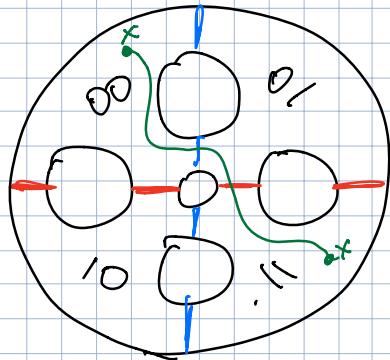
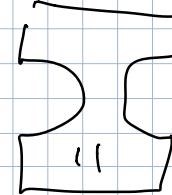
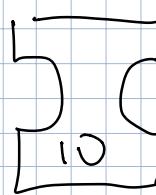
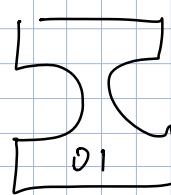
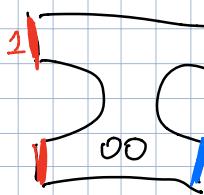
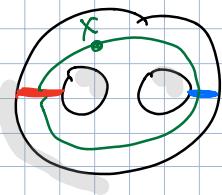
Subproblem: Find shortest cycle γ' s.t. $[\gamma'] = [\gamma]$

\mathbb{Z}_2 -homology cover

$$\bar{V} = \{(v, h) \mid v \in V \text{ and } h \in \mathbb{Z}^{2g}\}$$

$$\bar{E} = \{(u, h), (v, h') \mid uv \in E \text{ and } h \oplus h' = [uv]\}$$

\Downarrow
 $h \oplus [uv] = h'$



MSSP \geq
 $2^{O(g)} n \log n$