

Maximum Flows / Homology again

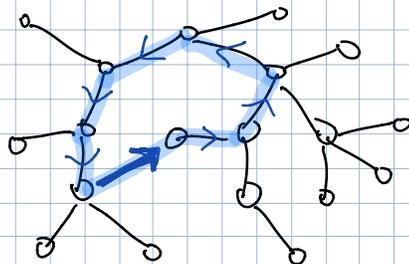
Homology with real coefficients:

Cycle space \mathbb{R}^{E-v+1} — elements are circulations

$$\phi: D \rightarrow \mathbb{R} \iff \partial\phi: V \rightarrow \mathbb{R} \text{ defined as}$$
$$\text{s.t. } \phi(d) = -\phi(\text{rev}(d)) \quad \partial\phi(v) = \sum_{u \rightarrow v} \phi(u \rightarrow v)$$

ϕ is a circulation iff $\partial\phi \equiv 0$

Basis: Fundamental cycles wrt spanning tree



Cut space \mathbb{R}^{F-1} — elements are Fractional cuts

Basis: Fundamental cuts wrt spanning tree

2-chain / Alexander #ing / face potentials $\alpha: F \rightarrow \mathbb{R}$

boundary $\partial\alpha: D \rightarrow \mathbb{R}$ defined as

$$\partial\alpha(d) = \alpha(\text{right}(d)) - \alpha(\text{left}(d))$$

ϕ is a boundary circulation $\iff \exists \alpha$ s.t. $\partial\phi = \alpha$

Boundary space: \mathbb{R}^{F-1}

Two circulations are homologous if $\partial\phi = \partial\phi'$

$$\iff \partial(\phi - \phi') \equiv 0$$

$\iff \phi - \phi'$ is a boundary

Homology space/group $H_1(\Sigma, \mathbb{R}) \cong \mathbb{R}^{(E-v+1) - (F-1)} = \mathbb{R}^g$

Basis: $\Gamma = \{ \text{cycle}(T, e) \mid e \in L \}$ for any tree-cotree (T, L, C)
 \uparrow directed arbitrarily

Every circulation is homologous with $\sum_{i=1}^{z_g} [\phi]_i \gamma_i$
 for some $[\phi] \in \mathbb{R}^{z_g}$

Dual basis: $\Lambda = \{ \text{cocycle}(C, e) \mid e \in L \}$
 \uparrow dir. arbitrarily

$$[u \rightarrow v]_i = \begin{cases} +1 & \text{if } u \rightarrow v \in \lambda_i \\ -1 & \text{if } v \rightarrow u \in \lambda_i \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{e \in \gamma} [e] = [\gamma] \leftarrow \text{homology class of } \gamma$$

$$[\phi] = \sum_e \phi(e) \cdot [e] \leftarrow \text{homology class of } \phi$$

$\uparrow \phi(d) = -\phi(\text{rev}(d))$ and $[d] = -[\text{rev}(d)]$
 so extra factor of 2? Whatever.

Yes these are the same vectors (up to x2)

ϕ is bdry iff $[\phi] = 0$

ϕ and ϕ' homologous iff $[\phi] = [\phi']$

Recall from planar flows:

Given a (not necessarily feasible) flow $F: E \rightarrow \mathbb{R}$

Find a feasible flow with the same ~~value~~ homology class.

Residual capacity $C_f(u \rightarrow v) = C(u \rightarrow v) - F(u \rightarrow v)$

\uparrow antisymmetric

Now find a feasible boundary circulation in the dual residual graph G_f^* .

Claim: