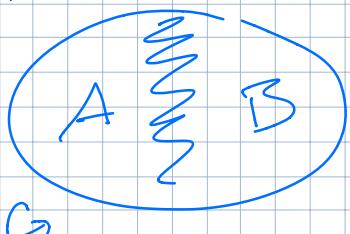


Project presentations during finals week

Reports due end of FW Dec 22?

ICES — Thursday

Separators

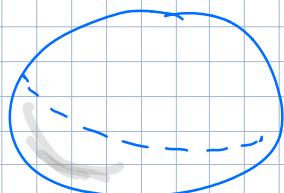


Planar: $\exists S \subseteq V \quad V \setminus S = A \cup B$

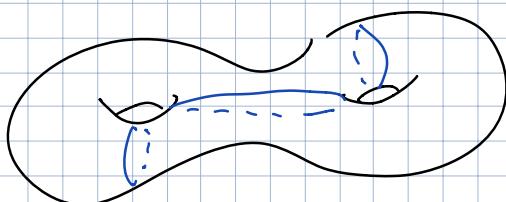
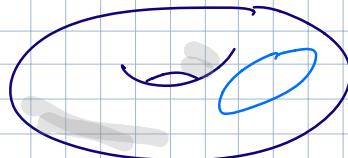
$$|S| = O(\sqrt{n})$$

$$|A| \leq 2n/3 \quad |B| \leq 2n/3$$

If G is a triangulation
then S is the vertices of
a cycle in G

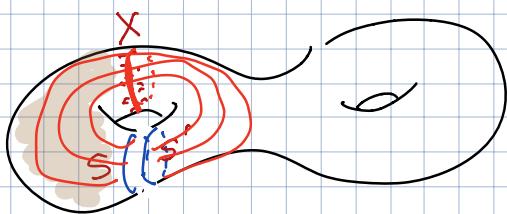


For graph on genus- g surface:
 $(\frac{1}{3}, \frac{2}{3})$ -separator of size $O(\sqrt{gn})$



There is a set of $\leq g$ cycles each of length $O(\sqrt{g} \log g)$
whose complement is planar

Lemma: Every ^{triangulation} graph has a non-sep cycle
of length $O(\sqrt{n})$.



Let $S =$ shortest nonsep cycle

Let $X =$ smallest set of vertices
separate S from S'

$$|X| \geq |S|$$

Menger's Theorem: $\max \# \text{disjoint paths} = \min \# \text{sep. vertices}$

$\Rightarrow \geq |S|$ disjoint paths from S to S'

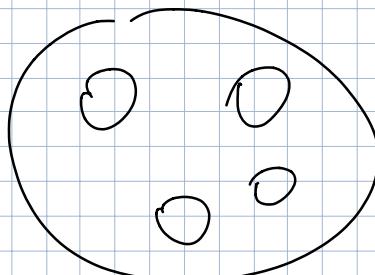
Each of these paths \Rightarrow non-sep cycle in G
 $\geq |S|$ vertices

$$n \geq |S|^2 \quad \square$$

Algorithm: $\text{for } i \leftarrow 1 \text{ to } g-1$
 Find shortest non-sep cycle γ_i in G
 $G \leftarrow G \setminus \gamma_i$

$O(g^2 \log n)$
~~2 approx: $O(n \log n)$~~ $\Rightarrow O(g \sqrt{n})$ vertices

In fact, shortest non-sep $O(\sqrt{\frac{n}{g}} \log g)$ $\hookrightarrow O(g \log g)$



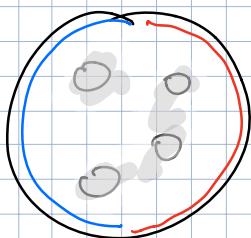
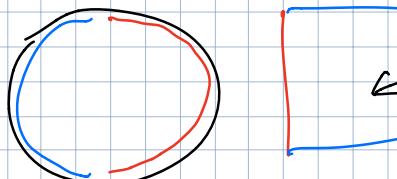
Boundary-to-boundary distances $O(g \log n)$

Bellman-Ford/Monge stuff

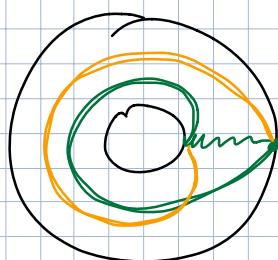
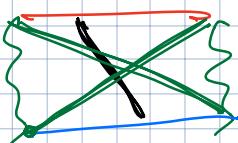
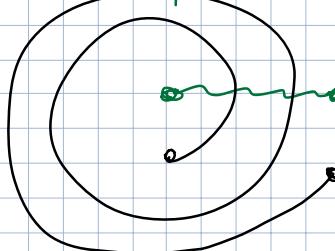
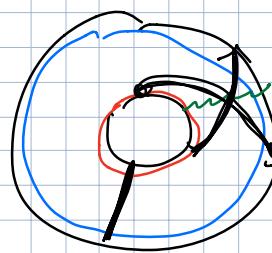
$$\hookrightarrow O(g) \times O(n \log n) + O(g^2) \times O(n)$$

Shortest paths
with neg edges

$$O(g^2 \log n)$$
 time

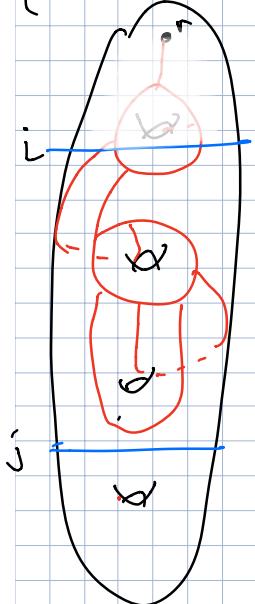


Shortest path between boundaries



Matrix of inner bdy to outer bdy
distances is the elementwise min
of 3 Monge matrices

[Gilbert-Hutchinson-Tarjan 84]



Greedy tree-cotree

$T = \text{BFS tree rooted at } r$

$C = \text{max spanning tree of } G^*$

where $w(e^*) = l(\text{loop}(T, e))$

$L = E \setminus T \cup C$

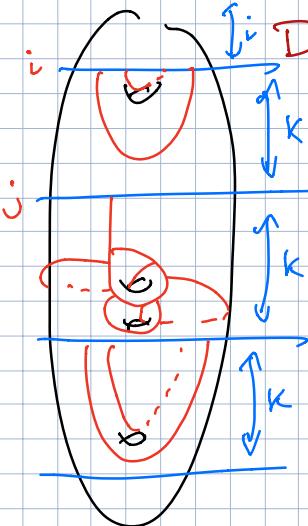
$R = \bigcup_{e \in L} \text{loop}(T, e) - \text{"reduced" cut graph}$

$$V_i = \{v \mid \text{dist}(r, v) = i\}$$

$G[i:j] - \text{induced by } \bigcup_{i \leq k \leq j} V_k$

$R[i:j] - \text{union of loops } \text{loop}(T, u) \cap G[:j]$
where $i \leq \text{dist}(r, u) \leq j$
 $i \leq \text{dist}(r, v) \leq j$

Lemma: $G[i:j] \setminus R[i:j]$ is planar



$$D(i, k) = \bigcup_{l: l \bmod k = i} V_l$$

For some i , $|D(i, k)| \leq n/k$

Each edge in L defines a path in

$G[i, i+k]$ of length $\leq 2k$

$$\boxed{D(i, k) \cup \bigcup_j T[jk+i, (j+1)k+i]}$$

Total #vertices

$$n/k + 2g(2k) = \boxed{\frac{n}{k} + 4gk}$$

$$n/k = \frac{5}{4}gk \rightarrow k^2 = \frac{n}{4g} \rightarrow k = \frac{1}{2}\sqrt{\frac{n}{g}}$$