Is $A$ homotopic to $B$?

Is $A \cdot B$ contractible?

$O(x+n)$ sort

$O(nk + k \log k)$ time

contractible!

$\delta_{k,x} \cdot \delta_{k,x} \cdot \partial_{k,x} \cdot \partial_{k,x} = \varepsilon$

reduced crossing sequence has length $\Omega(nk)$

$O((n+k) \log n)$ with correct preprocessing
Special case: Few self-intersections

- Trapezoidal decomposition
  - Extend walls up/down from vertices of \( P \)
  - obstacles \( O \)
  - self-intersection points
  - Sweep-line [Bentley-Ottman]

- Move vertical line \( \rightarrow \)

- Maintain sorted sequence of intersections (indices)
- Maintain a priority queue of potential future self-intersections + obstacles + vertices of \( P \)

- Each event \( \rightarrow O(1) \) ins/del in intersection seq
  - balanced binary tree \( \rightarrow O(\log n) \)
  - \( + O(1) \) ins/del in event queue \( \rightarrow O(\log n) \)

\[ \#\text{events} = n + k + s \]
\[ \#\text{self-intersections} \Rightarrow O((n+k+s) \log n) \text{ time} \]
vertical and horizontal ranking.

Goal:
Replace edge $\rightarrow$
vertex $\rightarrow$

horizontal rank of vertex $= 2 \times \# \text{obstacles to left}$
obstacle $= 2 \times \# \text{obstacles to left} + 1$

Vertical ranking:

$\sigma \uparrow \iota$
"$\sigma$ is above $\iota$"

$\sigma \uparrow \iota$ is the transitive closure of $\sigma \uparrow \iota$

For the sake of arguments pose

$\sigma_1 \uparrow \sigma_2 \uparrow \sigma_3 \ldots \sigma_2 \uparrow \sigma_1$

$\downarrow l = 1 \times$ $l = 2$

Leftmost-right end

$\sigma_1 \uparrow \sigma_2 \downarrow \sigma_2 \Rightarrow \sigma_2 \uparrow \sigma_2$

contradicting
Define fine vertical ranks for edge fragments and obstacles.

From any topological order of the dag →

\[ \text{vert rank obstacle} = 2 \# \text{obstacles with lower fine rank} + 1 \]

\[ \text{Fragment} = 2 \# \text{obst} \text{ with lower fine v. rank} \]

replace obstacle \((x, y) \rightarrow (h \text{ rank}, v \text{ rank})\)

vertex \(q\) \((h \text{ rank}(q), v \text{ rank}(pq))\)

edge \(pq\) \((h \text{ rank}(p), v \text{ rank}(pq))\)

\((h \text{ rank}(q), v \text{ rank}(q))\)

Represent \(P\) rectified polygon by alternating \(x\)- and \(y\)-coordinates.
Same crossing sequence

Remove zero-length edges
To simplify rectified polygon $\mathcal{P}$, slide brackets.

Store $k$ points $\mathcal{O}$.

Answer queries:

- Given a bracket $(x, y, x)$
- How far can we slide it?

$O(\log k)$ time

$\# \text{slides} \leq O(n+s)$